# MATH 113 Midterm 1 Partial solutions

The following gives the main solution ideas to some of the less straightforward problems that were graded. This does *not* constitute a complete solution/proof. I encourage you to fill in the details whenever necessary, or to ask me.

3. For part (i): No two elements of $G$ share an inverse. Thus, suppose that $a_1, a_2, \ldots, a_{2n-1}$ are the $2n-1$ elements of $G$ not equal to the identity. By pairing elements with their inverses, we see that at least one element, say, $a$, is left without a pairing. That is, it must be self-inverse $a = a^{-1}$. That is $a^2 = e$.

Part (ii) was cooked. You need to further assume $G$ is abelian. In that case the proof is as follows: suppose $a$ and $b$ are two elements whose square is the identity. Then observe that $\langle a, b \rangle$ is a four element group, consisting of $e, a, b, ab = ba$. But then $G$ has a subgroup of order 4 while 4 doesn't divide $2n$, a contradiction.

4. Let $a\ G \setminus \{e\}$. Then the cyclic group $\langle a \rangle$ is all of $G$. Thus $G$ is cyclic. Now any infinite cyclic group is isomorphic to $\mathbb{Z}$. We know $\mathbb{Z}$ has all kinds of nontrivial, proper subgroups. So $G$ must be finite. Not use the fundamental theorem of finitely generated abelian groups to conclude that $G$ is of prime order (another way is to use Cauchy's theorem).

5. This is a problem in trying to work through the definition. What is being asked is to show that the stablizer, denoted say $(G)_{gH}$ is (1) contained in $gHg^{-1}$ and (2) contains $gHg^{-1}$. Let's do (1): let $h \in G_{gH}$. Then $h \star (gH) = (hg)H = gH$ for all $g \in G$. Thus $g^{-1}hg \in H$. Hence $h \in gHg^1$. Try to prove (2).

6. This is another problem in properly understanding the question. After that, (i) and (ii) are routine. For (iii) note that we are asking for the kernel of $\Phi$, which means that we want all $g$ such that the automorphism defined by $h \mapsto ghg^{-1}$ is identically the identity automorphism. That is $ghg^{-1} = h$ for all $h \in G$. This is the *center* $Z(G)$ of $G$. For (iv) you have to get your hands dirty and actually show that the image is normal by conjugating each element in the image by an automorphism element and showing that the result is one of those automorphisms from (ii).