Midterm Exam #2

Math 113 - Section 002 - Instructor: Jameel Al-Aidroos
Tuesday, August 1 - 12:10-1:30 (80 Minutes)

Full Name: ______________________________

SID: ______________________________

Directions:

- Do not open your exam until your are instructed to do so.
- No notes, texts or calculating devices of any kind may be used during this exam.
- Unless you are specifically instructed otherwise, you must show all your work, and justify all claims. Answers without explanation will not receive credit.
- Write your solutions in the space provided. If necessary, use the backs of the pages or the extra pages attached to your exam, and *write a clear note on the question page* indicating that you have done so.
- When time is called, you must stop working and close your exam.

This exam has 7 pages including this one and one (almost) blank page.

Good Luck!

Question	Grade
#1.	/35
#2.	/20
#3.	/25
#4.	/20
Total	/100

Question #1 has 5 parts. Each part requires a few lines justification, or a short calculation. For True or False questions, please indicate either TRUE or FALSE, and give a brief explanation. You should aim to spend no more than 30 minutes on question #1.

1. (a) *(7 points)* True or False? $x^5 + 30x^4 + 60x^2 + 15x + 150$ is irreducible in $\mathbb{Q}[x]$.

   (b) *(7 points)* Write down two rings that have the same cardinality (number of elements) but that are non-isomorphic. Prove they are not isomorphic.

   (c) *(7 points)* True or False? There are exactly four groups of order 36.

(d) *(7 points)* There is a group G of order $168 = 2^3 \cdot 3 \cdot 7$ with the property that G does not have any non-trivial normal subgroups. (I.e. G is a *simple* group.) What can you say about the number of Sylow-7 subgroups of G?

(e) *(7 points)* Let R be a commutative ring with $1 \in R$. Show that a zero-divisor in R can't have a multiplicative inverse.

Questions #2 -#4 will be graded for correctness and for the clarity of your explanations. Justify all your work.

2. Let R and S be commutative rings, and let $\phi : R \to S$ be a homomorphism of rings. Let I be an ideal of R.

   (a) *(10 points)* Prove: If ϕ is onto then the image $\phi(I)$ is an ideal in S.

   (b) *(10 points)* Give an example to show that the statement in (a) is not true if we omit the assumption that ϕ is onto.

3. Let G be a finite group, and let H be a subgroup of G.

   (a) *(5 points)* What is the definition of 'G acts on a set S'?

   (b) *(5 points)* Define 'the normalizer of H in G'.

   (c) *(15 points)* Suppose p is the *smallest* prime divisor of $|G|$, and suppose that $|G|/|H| = p$. Prove that H is a normal subgroup of G. [Hint: Let the group H on the set G/H by multiplication, and examine the sizes of the orbits.]

4. Let R be a commutative ring with unity. A proper ideal $I \subset R$ is called *prime* if it satisfies the condition: for all $a, b \in R - I$, we have $ab \in R - I$

(a) *(15 points)* Show that an ideal $I \subset R$ is prime if and only if the quotient ring R/I is an integral domain with unity.

(b) *(5 points)* Show that every maximal ideal of R is a prime ideal of R.

This page is intentionally (almost) blank.
You may use it for scrap work.