Midterm Exam #2

Math 113 - Section 001 - Instructor: Jameel Al-Aidroos
Tuesday, July 31 - 12:10-1:40 (90 Minutes)

Full Name: ______________________________

SID: ______________________________

Directions:

- Do not open your exam until your are instructed to do so.
- No notes, texts or calculating devices of any kind may be used during this exam.
- Unless you are specifically instructed otherwise, you must show all your work, and justify all claims. Answers without explanation will not receive credit.
- Write your solutions in the space provided. If necessary, use the backs of the pages or the extra pages attached to your exam, and *write a clear note on the question page* indicating that you have done so.
- When time is called, you must stop working and close your exam.

This exam has 8 pages including this one and one (almost) blank page.

Good Luck!

Question	Grade
#1.	/40
#2.	/20
#3.	/20
#4.	/20
Total	/100

Question #1 has 5 parts. Each part requires a few lines justification, or a short calculation. For True or False questions, please indicate either TRUE or FALSE, and give a brief explanation. You should aim to spend no more than 30 minutes on question #1.

1. (a) *(5 points)* True or False? $x^3 + 15x^2 + 17x + 29$ is irreducible in $\mathbb{Q}[x]$.

   (b) *(5 points)* Let G be the symmetry group of the tetrahedron. How many elements of order 3 are there in G?

(c) *(5 points)* Give a complete description of the group D_7.

(d) *(5 points)*

i. True of False? Every integral domain is a field.

ii. True or False? Every field is an integral domain.

(e) *(20 points)* For parts (i) to (v), NO explanation is required.

Give an example of:

i. a non-commutative ring

ii. a ring R that contains exactly 4 invertible elements

iii. two non-isomorphic rings that are isomorphic as groups

iv. three non-isomorphic Principal Ideal Domains

v. five non-isomorphic groups of order 24

Questions #2 -#4 will be graded for correctness and for the clarity of your explanations. Justify all your work.

2. Let $\phi : R \to S$ be a homomorphism of rings.

   (a) *(10 points)* Show that the kernel of ϕ is an ideal of R.

   (b) *(10 points)* Suppose ϕ is invertible and $\psi : S \to R$ is its inverse. Show that ψ is a ring homomorphism.

3. Consider the action of a group G on a set S.

   (a) *(5 points)* Define the terms 'Orbit' and 'Stabilizer'.

   (b) *(15 points)* Let $|G| = p^n m$, where p is prime and $(p, m) = 1$. Let H be a Sylow-p-subgroup of G, and let H act on G/H by multiplication. Show that there is at least one orbit of size 1.

4. Consider the ring $R = \mathbb{Z} \times \mathbb{Z}$.

   (a) *(7 points)* Let H be the *cyclic subgroup* generated by the element $(1, 3) \in R$. Show that there is an isomorphism of groups $R/H \cong \mathbb{Z}$.

   (b) *(5 points)* Is the isomorphism from the first part an isomorphism of rings?

   (c) *(8 points)* Now let I by the *ideal* generated by the element $(1, 3) \in R$. Show that I is a maximal ideal.

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You may use it for scrap work.