Midterm Exam #1

Math 113 - Section 002 - Instructor: Jameel Al-Aidroos
Tuesday, July 11 - 12:10-1:30 (80 Minutes)

Full Name: ____________________

SID: ____________________

Directions:

- Do not open your exam until your are instructed to do so.
- No notes, texts or calculating devices of any kind may be used during this exam.
- Unless you are specifically instructed otherwise, you must show all your work, and justify all claims. Answers without explanation will not receive credit.
- Write your solutions in the space provided. If necessary, use the backs of the pages or the extra pages attached to your exam, and *write a clear note on the question page* indicating that you have done so.
- When time is called, you must stop working and close your exam.

This exam has 9 pages including this one, the formulas, and one (almost) blank page.

Good Luck!

Question	Grade
#1.	/35
#2.	/25
#3.	/15
#4.	/10
#4.	/15
Total	/100

Question #1 has 7 parts. Each part requires only a line or two of justification, or a short calculation. For True or False questions, be sure to indicate either TRUE or FALSE, and give a very brief explanation.
You should aim to spend no more than 30 minutes on question #1.

1. (a) *(5 points)* Find $(50, 63)$, and find integers, m and n such that $(50, 63) = m \cdot 50 + n \cdot 63$

   (b) *(5 points)* Let $\sigma = (1\,3\,7)(2\,8\,7\,3)(2\,4\,1\,6) \in S_8$. Write σ, and σ^{-1} in disjoint cycle notation.

   (c) *(5 points)* True or False? For any group, G, with $|\,G| = 17$, the group G must be abelian.

(d) *(5 points)* True or False? The function $\phi : \mathbb{Z}/7\mathbb{Z} \to \mathbb{Z}/4\mathbb{Z}$ given by $\phi([k]_7]) = [k]_4$ is well-defined.

(e) *(5 points)* True or False? In a group G, for any $a, b \in G$, there is a unique $x \in G$ satisfying the equation $ax = b$.

(f) *(5 points)* True or False? In an abelian group G, for any $a, b \in G$, $o(ab) = o(a)o(b)$.

(g) *(5 points)* True of False? For a finite group G and a non-trivial normal subgroup N, let G/N be the quotient group. Then $|\,G/N| = |\,G|$.

Questions #2 -#4 will be graded for correctness and for the clarity of your explanations. Justify all your work.

2. (a) *(5 points)* Define "group homomorphism".

   (b) *(5 points)* Define "kernel of a group homomorphism"

   (c) *(10 points)* Let $\phi : G \to G'$ be a homomorphism. Show that $\ker(\phi)$ is a subgroup of G.

   (d) *(5 points)* Show that $\ker(\phi) \lhd G$.

3. Let $(G, \cdot)$ be a group. Let G^{op} be the same set as G, and define an operation $*$ on G^{op} by $a * b = b \cdot a$ for all $a, b \in G^{op}$.

   (a) *(10 points)* Show that $(G^{op}, *)$ is a group.

   (b) *(5 points)* Show that $\phi : G \to G^{op}$ defined by $\phi(g) = g^{-1}$ is an isomorphism.

4. Let G and G' be finite groups, and let $\phi : G \to G'$ be a group homomorphism. Let $\phi(G)$ be the image of the homomorphism.

   (a) *(5 points)* Show that $|\,\phi(G)|$ divides $|\,G'|$.

   (b) *(5 points)* Prove that $|\,\phi(G)|$ divides $|\,G|$.

5. (a) *(5 points)* Let H and K be normal subgroups of a group G. Suppose $H \cap K = \{e\}$. Show that for any $h \in H$ and $k \in K$, we have $hk = kh$. [Hint: consider $hkh^{-1}k^{-1}$.]

(b) *(5 points)* Let H and K be subgroups of G with $|H| = 3$ and $|K| = 5$. Show that $H \cap K = \{e\}$. [Hint: What can you say about the orders of the elements of H? And what about the elements of K?]

(c) *(5 points)* Let H and K be normal subgroups of G with $|H| = 3$ and $|K| = 5$ and $|G| = 15$. Show that $G \simeq \mathbb{Z}/15\mathbb{Z}$.

This page is intentionally (almost) blank.
You may use it for scrap work.