

FINAL Exam

Math 113 - Section 002 - Instructor: Jameel Al-Aidroos
Thursday, August 17 - 12:11-1:59 (108 Minutes)

Full Name: _____

SID: _____

Directions:

- Do not open your exam until you are instructed to do so.
- No notes, texts or calculating devices of any kind may be used during this exam.
- Unless you are specifically instructed otherwise, you must show all your work, and justify all claims. Answers without explanation will not receive credit.
- Write your solutions in the space provided. If necessary, use the backs of the pages or the extra pages attached to your exam, and *write a clear note on the question page* indicating that you have done so.
- When time is called, you must stop working and close your exam.

This exam has 8 pages including this one and one (almost) blank page.

Good Luck!

Question	Grade
#1.	/35
#2.	/25
#3.	/10
#4.	/10
#5.	/10
#6.	/10
Total	/100

Question #1 has 7 parts. Each part requires only a clever line or two of justification, or a short calculation. For True or False questions, be sure to indicate either TRUE or FALSE, and give a very brief explanation. You should aim to spend no more than 35 minutes on question #1.

1. (a) (5 points) TRUE or FALSE? If a is an element of a finite group G , then the order of a divides the order of G .

(b) (5 points) TRUE or FALSE? Every algebraic field extension of \mathbb{Q} is a subfield of \mathbb{R} .

(c) (5 points) Name seven non-isomorphic groups of order 600.

(d) (5 points) Let R be a commutative ring with unity, and let M be a maximal ideal in R . Let a be an element of R that's not in M . Show there's an $x \in R$ and an $m \in M$ with $ax - m = 1$.

(e) (5 points) Show that $\langle x^3 + x + 1 \rangle \cap \langle x - 2 \rangle$ is a principal ideal of $\mathbb{F}_7[x]$.

(f) (5 points) What is $[\mathbb{Q}(\sqrt[3]{5}, \sqrt[4]{3}) : \mathbb{Q}]$?

(g) (5 points) TRUE or FALSE: If G is any group of order 31, then there's a finite field \mathbb{F} and a group isomorphism $\phi : G \rightarrow \mathbb{F}^\times$.

Questions #2 -#5 will be graded for correctness and for the clarity of your explanations. Justify all your work.

2. Let $\phi : R \rightarrow S$ be a homomorphism of rings.

(a) (5 points) Define the terms "ideal" and " $\ker(\phi)$ ".

(b) (10 points) Show that the kernel of ϕ is an ideal.

(c) (10 points) Suppose R is a field. Show that the homomorphism $\phi : R \rightarrow S$ is either one-to-one or the zero homomorphism.

3. (10 points) Let G be a group, and let G act on a set S . (Denote the action of an element $g \in G$ on an element $s \in S$ by gs .) For elements s_1 and s_2 , define the relation:

$$s_1 \sim s_2 \text{ if and only if } s_1 \in \text{Orb}(s_2)$$

Show that \sim is an equivalence relation on S by verifying the axioms.

4. (10 points) Let G be a group with $|G| = 2^2 \cdot 5$. How many elements of order 5 does G have?

5. (10 points) Let $E \subset F$ be an extension of fields. Suppose that for some prime, p , we have $[F : E] = p$. For any $\beta \in E$ with $\beta \notin F$, show that $F(\beta) = E$

6. (10 points) Show that there exists an isomorphism of the two quotient rings:

$$\mathbb{F}_3[x]/\langle x^3 + x^2 + 2x + 1 \rangle \quad \text{and} \quad \mathbb{F}_3[x]/\langle x^3 + 2x^2 + 1 \rangle$$

This page is intentionally (almost) blank.
You may use it for scrap work.