MATH113 PROBLEMS: HOMEWORK 2 (DUE TUESDAY, JULY 3)

(1) **§2.1 #1** For each of following sets G with the operation $*$ indicated, either show that $(G, *)$ is a group, or find a group axiom that $(G, *)$ fails to satisfy.
- **(a)** $G = \mathbb{Z}$, and $a * b = a - b$
- **(b)** $G = \mathbb{Z}$, and $a * b = a + b + ab$
- **(d)** $G = \mathbb{Q} - \{-1\}$, and $a * b = a + b + ab$
- **(f)** G is the set of all rational numbers that have denominator divisible by 5, when written in lowest terms, and $a*b = a+b$, the usual addition of rational numbers.

(2) Fix two real numbers, $a, b \in \mathbb{R}$. Let $G = \{f : \mathbb{R} \to \mathbb{R} \mid f(a) = b\}$, and let G have the usual pointwise addition of functions. What are the minimum restrictions on a and b to ensure that G is a group?

(3) **§2.1 #16** Let G be a group with the property that for every $g \in G$, $g^2 = e$. Show that G must be abelian.

(4) **§2.2 #3** Let G be a group with the following property: there exist three consecutive positive integers i such that for all $a, b \in G$, $(ab)^i = a^i b^i$. Prove that G must be abelian.

(5) **§2.4 #1(a)** Let $S = \mathbb{R}$, and consider the relation: $a \approx b \iff a - b$ is rational. Show that $\approx$ is an equivalence relation.

(6) **§2.3 #3** Let S_3 denote the symmetric group of degree 3. Find all the subgroups of S_3. Use cycle notation. You need not show your work.

(7) Choose a subgroup A of order 3, and another B of order 2. Write down all the left and right cosets of A in S_3. Notice that for any $\sigma \in S_3$, $\sigma A = A\sigma$. Now write down all the left and right cosets for B. What do you notice?

(8)
- (a) **§2.3 #16** If a (non-trivial) group G has no proper subgroups, prove that G is cyclic of order p, where p is a prime number.
- (b) Prove the converse of the statement in (a).

(9) **§2.4 #14** List all the elements of U_{20}; find their orders. Is U_{20} cyclic?