

Math 113 Homework # 5, selected solutions

Fraleigh 10.39: Suppose that H is a subgroup of index 2 in a finite group G , so that $|H| = |G|/2$. Then there are only two left cosets, and one of the left cosets is H , so since G is the disjoint union of the left cosets, the other left coset is $G \setminus H$. Since $|H| = |G|/2$, there are also only two right cosets, and since one of the right cosets is H , the other right coset is $G \setminus H$. Thus the left and right cosets are the same.

Fraleigh 10.40: Let k be the order of a , which is also the order of the cyclic subgroup generated by a . By Lagrange's theorem, k divides n , so we can write $n = kd$ where d is an integer. Then $a^n = a^{kd} = (a^k)^d = e^d = e$.