

1a) Fill in the blanks (2 points  $\times$  10 blanks = 20 points):

$\theta$ in degrees	$\theta$ in radians	$\sin \theta$	$\cos \theta$
$30^\circ$			
	$\frac{\pi}{4}$		
$240^\circ$			
$0 < \theta < 90^\circ$	$0 < \theta < \frac{\pi}{2}$	$\frac{3}{5}$	

1b) (5 points) Simplify the following expression:

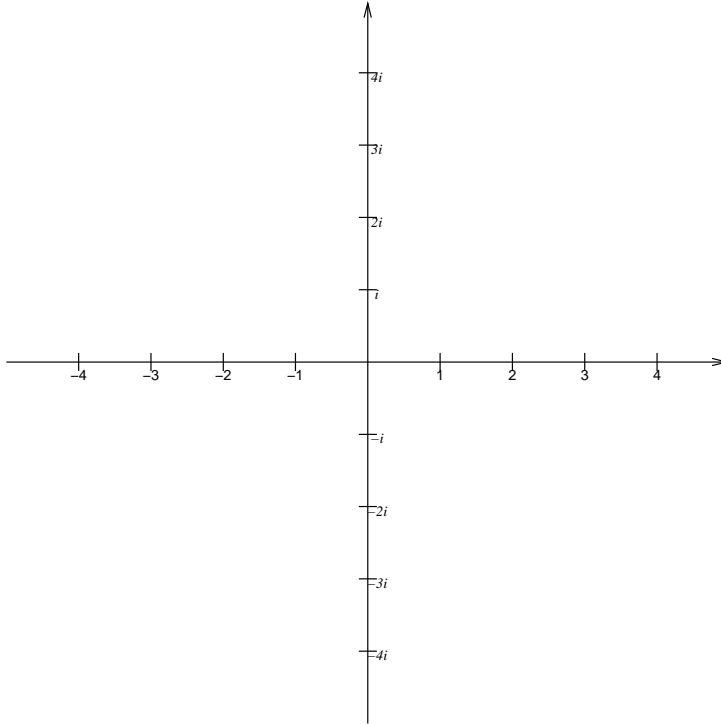
$$\sin^2(\pi - 2\beta) + \cos^2(\pi - 2\beta)$$

Page 2

2a) (5 points) What are the amplitude and period of the function

$$y = 2 \cos\left(\frac{\pi}{3}x - \pi\right)?$$

2b) (4+6 points) Plot and label the following complex numbers in the complex plane: A)  $2 - 3\sqrt{-1}$  and B)  $2\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$ .



2c) (5+5 points) Compute the product of the two complex numbers

$$(\cos \alpha + i \sin \alpha)(\cos \beta + i \sin \beta)$$

in **two different ways**: both by multiplying it out, and by addition of angles.  
(This is how we derived the angle addition formulas for cosine and sine.)

3a) (4 points) Add the fractions  $\frac{\pi}{3} + \frac{-\pi}{4}$  and simplify.

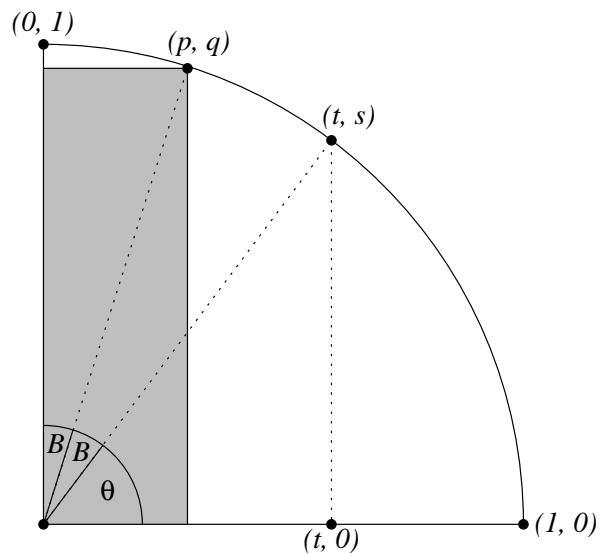
3b) (8 points) Compute  $\cos \frac{\pi}{12}$  and simplify your answer. (The previous question was a hint.)

3c) (8 points) Compute  $\sin \frac{\pi}{12}$  and simplify your answer.

3d) (5 points) Now use the double-angle formula for sine to compute  $\sin(2\frac{\pi}{12})$ . Simplify your answer. (If your answer doesn't agree with the value for  $\sin \frac{\pi}{6}$ , then you've made a mistake somewhere on this page.)

4a) (6 points) Write the angle addition formulas for  $\cos(A + B)$  and  $\cos(A - B)$ . Add the two equations and obtain a product-to-sum formula.

The figure below shows the first quadrant of the unit circle in the  $(x, y)$ -plane, with various points and angles labeled. Notice that  $(t, s)$  is directly above the point  $(t, 0)$ , and that  $(p, q)$  is exactly halfway around the arc from  $(t, s)$  to  $(0, 1)$ .



4b) (9 points) Fill in the missing angles:

$$t = \cos(\text{_____}) \quad p = \cos(\text{_____}) \quad q = \cos(\text{_____})$$

4c) (10 points) What is the area of the shaded rectangle in terms of  $t$ ? You should simplify your answer to get rid of any angles or variables other than  $t$ .

**Hint:** Use your answers to 4a) and 4b).