Miscellaneous practice problems, 16 Dec 2000

1. Which of the following is an antiderivative for xe^x?
a) $e^{x^2/2}$
b) $\frac{x^2}{2}e^x$
c) $\frac{x}{x+1}e^{x+1}$
d) $(x-1)e^x$

2. Use the Fundamental Theorem of Calculus to evaluate the following limit:

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2\cdot\frac{1}{n}$$

(Hint: What function and interval is this a Reimann sum for? What happens as n goes to infinity?)

3. Use the fact that $\frac{d}{dx}(\cos(x)) = -\sin(x)$ to find the area of the region above the x-axis and below the graph of $y = \sin(x)$ over the interval $[0, \pi]$.

4. A company can make anywhere between 0 and 100 gadgets at a total cost of $3q - \frac{q^3}{30000}$ dollars, where q is the quantity produced. They can sell 100 gadgets at a price of \$2.75 per gadget; every \$.10 increase in price decreases the quantity they can sell by 10. How many gadgets should the company make (and sell) in order to maximize profit? Where should they set the price? Justify your answer.

5. Suppose $f(x)$ is an antiderivative of e^{-x^2}, and $f(0) = 0$.
(a) When is $f(x)$ increasing?

(b) When is $f(x)$ concave upwards?

(c) Is $f(-1)$ positive or negative?

(d) At what points does $f(x)$ have local maxima or minima?

(The problems on this page were handed out at Saturday's review session.)

T/F: $\int_a^b f'(t)\,dt = f(b) - f(a)$
T/F: $a \ln b = \ln(ba)$
T/F: $\ln b - \ln a = \ln(ba)$
T/F: $\ln b + \ln a = \ln(ba)$

Review stuff (quick). This stuff should be easy for you. It's necessary as part of solving more complicated problems:
(Chapters numbers in **bold** are from the Goldstein text)
0.4 2, 17, 26, 34
$e^x e^h = ?$ $\quad e^{\ln a} = ?$ $\quad \ln e^a = ?$ $\quad \ln 1 = ?$ $\quad e^0 = ?$

Derivatives:
1.3 4, 14, 16, 23, 24, 48, 52
1.4 2, 4, 32 (Use definition of derivative)
1.5 7–10
1.6 23, 32, 34, 43
1.7 4, 6, 13
3.1 2, 4, 18, 24, 28

A couple applications of the derivative:
2.1 40
2.2 6, 23, 28
2.4 9, 32
2.6 1, 3
2.7 17

Exponential and compound growth:
5.1 9
5.2 9, 12

How many a) local minima, b) local maxima, c) global minima, and d) global maxima do the following functions have?
i) $\sin(x)$, defined on all real numbers
ii) $\sin(x)$, defined on the interval $[0, 2\pi]$
iii) x^3 on $[-1, 1]$
iv) x^3 on all $\mathbb{R}$
v) e^x on all $\mathbb{R}$
vi) x^2 on $[-1, 1]$
vii) x^2 on all $\mathbb{R}$
viii) $x(x-2)(x+2)$ on all $\mathbb{R}$