

Qualifying Examination Syllabus

Guoliang Wu

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Time: Monday, May 8, 2006, 11-2pm Location: 961 Evans

Major Topic: Partial Differential Equations

Laplace's Equation: Fundamental solution; mean-value formulas; properties of harmonic functions; Green's function for the half-space and ball; Dirichlet's principle.

Heat Equations: Fundamental solution; initial-value and nonhomogeneous problems; mean-value formula; regularity; energy methods.

Wave Equation: d'Alembert's formula; solution by spherical means ($n = 2, 3$); Huygen's principle; nonhomogeneous problem, energy methods.

First-Order Equations: Complete integrals; envelopes; characteristic ODE; local existence theorem.

Hamilton-Jacobi Equations: Hamilton's ODE; Legendre transform; Hopf-Lax formula; viscosity solutions, uniqueness(statement only); control theory and dynamic programming; Hamilton-Jacobi-Bellman equation.

Scalar Conservation Laws: Shocks; entropy condition; Lax-Oleinik formula; weak solutions, uniqueness; Riemann's problem; Decay in L^∞ -norm.

Sobolev Spaces: Hölder spaces; Weak derivatives; definition of Sobolev spaces; approximation by smooth functions; extensions; traces; inequalities of Gagliardo-Nirenberg-Sobolev, Morrey and Poincare; compact embeddings; difference quotients; differentiability a.e.; Fourier transform characterization of $H^k(\mathbb{R}^n)$; the space H^{-1} .

Second-Order Elliptic Equations: Definitions of elliptic equations and weak solutions; existence theorems for weak solutions; regularity; maximum principles; eigenvalues and eigenfunctions.

Miscellaneous Techniques: Separation of variables; similarity solutions; Fourier transform methods; Hopf-Cole transformation; Hodograph and Legendre transforms; monotonicity methods; fixed point methods; method of subsolutions and supersolutions.

Calculus of Variations: First and second variation; Euler-Lagrange equation; existence of minimizers; regularity; constraints; critical points.

Reference: Evans, *Partial Differential Equations*.

Related Topic: Real and Functional Analysis

Measure Theory and Real Analysis: Properties of measures, measurable functions and integrals; Lusin's Theorem; Egoroff's Theorem; Fatou's Lemma; Dominated Convergence Theorem; convergence in measure; Fubini's Theorem; differentiation of Radon measures; Lebesgue Decomposition Theorem; Lebesgue-Besicovitch Differentiation Theorem.

Definition and elementary properties of Hausdorff measure; Hausdorff dimension; Hausdorff measure and Lipschitz functions; Linear maps and Jacobians; Area Formula and Coarea Formula (statement); Change of variables formula.

Functional Analysis: Normed linear spaces; Hahn-Banach Theorem; open mapping theorem and closed graph theorem; compact operators; Fredholm Theory; Fourier transform; Plancherel's Theorem; Hilbert spaces.

References: Evans, Gariepy, *Measure Theory and Fine Properties of Functions*; Folland, *Real Analysis*; Evans, *Partial Differential Equations*.

Minor Topic: Numerical Solutions of Differential Equations:

Ordinary Differential Equations: Basic theory: convergence, consistency, stability, stiffness, A -stability; Single-step methods: Euler, trapezoidal, midpoint; Runge-Kutta methods; Legendre polynomials, Gaussian quadrature.

Partial Differential Equations (Finite Volume Method) Convergence, stability and consistency; CFL condition; upwind methods; Lax-Wendroff method; Lax-Friedrichs method; REA algorithm, Godunov's Method.

References: *Lecture notes from J. Strain for Math 228*; R. J. LeVeque, *Finite Volume Methods for Hyperbolic Problems*.