

MATH 53 – PRACTICE MIDTERM #1

Problem #1 (a). Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x, y) = (x^2 + y)^2$.

(b). Compute ∇g for $g(x, y, z) = \sin(xy^2z^3)$.

Problem #2. Show that the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^8 + y^5}{x^8 + 7y^4}$$

does not exist.

Problem #3. Find the length of the curve given in polar coordinates by the formula

$$r = 1 - \cos \theta \quad (0 \leq \theta \leq 2\pi).$$

(Hint: $1 - \cos \theta = 2 \sin^2(\frac{\theta}{2})$.)

Problem #4. Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ where z is implicitly defined as a function of x and y by the formula

$$yz + \ln z = x^2 + y^3.$$

Problem #5. The surfaces determined by the equations

$$x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0 \quad \text{and} \quad x^2 + y^2 + z^2 = 11$$

intersect along a curve C that passes through the point $(1, 1, 3)$. Find a tangent vector to the curve C at $(1, 1, 3)$.

Problem #6. Assume $u = x - y, v = y - z, w = z - x$ in the function $f(u, v, w)$. Show that

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0.$$

Problem #7 (a). Let $\mathbf{r}(t)$ be a 3-dimensional vector function which satisfies

$$\mathbf{r}'' = |\mathbf{r}| \mathbf{r}$$

for all times t , where $' = \frac{d}{dt}$. Show that the vector $\mathbf{n} = \mathbf{r}' \times \mathbf{r}$ does not change with time.

(b). What is $\mathbf{n} \cdot \mathbf{r}$?

(c). Deduce that the curve traced out by \mathbf{r} lies in a plane through the origin.