Practice Writing Proofs. Math 113 Summer 2014.

These problems are to help you get some practice writing simple proofs, especially in preparation for the first exam. The proofs should all be fairly short.

Cautions/Suggestions:

1. Do not assume what you're supposed to be proving.
2. Each time you use a new variable (g, H, etc.), make sure you first say clearly where it lives. For example: "Let g be an element of G" or "pick any subgroup H of G"...
3. Keep your attention focused on what you want to prove - this should direct your steps, but always start from a) what's given, or b) a fact that's known from class.
4. The first and last lines are in some sense the most important, as they dictate the "logical flow" of the proof - where you start from, and what you're proving.
5. Make sure your work is understandable, coherent and well-presented - if the grader has to work to determine what/where your solution is, they will get grumpy and more prone to giving lower grades.

Problems:

1. If H and K are subgroups of a group G, prove that $H \cap K$ is also a subgroup of G.
2. If $f\colon G \to H$ is a homomorphism, prove that $\ker f$ is a normal subgroup of G.
3. A coset gH of a subgroup H of G is equal to H if and only if $g \in H$.
4. Prove that any group G whose order is a prime number p is cyclic.
5. Prove that the function $f\colon \mathbb{Z}/3\mathbb{Z} \to S_3$ given by $f(\overline{n}) = (123)^n$ is a) well-defined; b) a homomorphism; and c) injective.
6. If G and H are groups, prove that $G \times H \cong H \times G$.
7. Let p be a prime number. Prove that $f\colon \mathbb{Z}/p\mathbb{Z} \to \mathbb{Z}/p\mathbb{Z}$ given by $f(\overline{n}) = \overline{n^p}$ is the identity map (you can assume that it's well-defined).
8. If H is a normal subgroup of G, then any conjugate gHg^{-1} of H is also normal.