Worksheet 7/8. Math 113 Summer 2014.

These problems are intended as supplementary material to the homework exercises and will hopefully give you some more practice with actual examples. In particular, they may be easier/harder than homework. Problems with an asterisk (*) should be more challenging than the rest.

1. Prove Corollary 9.1.3, namely that if $|G| = p^r m$, with $p \not| m$, and k_p is the number of Sylow p-subgroups of G, then $k_p | m$. (note: this result can be very useful in applications of the Sylow theorems, e.g., the next problem).
2. Prove that a group of order 42 cannot be simple.[1]
3. Prove that a group of order 96 cannot be simple.
4. Prove the result from lecture used in the proof of SYL1 that $\binom{n}{p^r} \not\equiv 0 \bmod p$, where $n = p^r m$, $m \neq 1$, $p \not| m$.
5. * Prove that a group of order 56 cannot be simple (though similar to problem 2, you have to be more careful with this one)

[1] Recall that G is **simple** if it contains no proper nontrivial normal subgroup.