Worksheet July 21. Math 113 Summer 2014.

These problems are intended as supplementary material to the homework exercises and will hopefully give you some more practice with actual examples. In particular, they may be easier/harder than homework. Problems with an asterisk (*) should be more challenging than the rest.

1. Prove parts 1 and 3 of lemma 1.2.2, namely that (1) $0a = a0 = 0$ and (3) $(-a)(-b) = ab$, for $a, b \in R$.

2. Prove that every nontrivial idempotent is a zerodivisor [hint: if e is idempotent, consider the element $1 - e$]

3. Find all the units and zerodivisors in the ring $\mathbb{Z}/6\mathbb{Z}[x]$ of polynomials with coefficients in $\mathbb{Z}/6\mathbb{Z}$.

4. * Let $R = \mathbb{C}[[x]]$ be the ring of formal power series with coefficients in $\mathbb{C}$.

   (a) Prove that $1 - x$ is a unit in R (hint: geometric series)

   (b) Prove that any linear (degree ≤ 1) polynomial p, such that $p(0) \neq 0$, is a unit in R.

   (c) Using the fact that every polynomial can be fully factored over $\mathbb{C}$, determine all the units in R.