Math 110, Summer 2012 : JCF review problems

Polynomials, representations

1. Determine the minimal polynomials of the following matrices:

- $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$,

- $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix}$,

- $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$,

- $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

Canonical form of an endomorphism

Determine the Jordan canonical form J of the above matrices. Find $P \in GL_n(\mathbb{C})$ such that $P^{-1}AP = J$.

Jordan canonical form

Suppose that you are given a matrix $A \in Mat_5(\mathbb{C})$ such that μ_A is one of the below polynomials. What are the possibilities for the Jordan canonical form of A?

- $\mu_A = (t+1)^2(t-2)^2$,

- $\mu_A = t(t+4)(t-2)$,

- $\mu_A = t^2(t-2)$,

- $\mu_A = (t-5)(t-1)(t+3)^3$.

(*Hint: recall that μ_A is an annihilating polynomial and we can use the Primary Decomposition Theorem to determine a direct sum decomposition of $\mathbb{C}^5$. What are the allowed blocks of the Jordan form of A?*)