Worksheet 7/30. Math 110, Summer 2012

*An asterisk * denotes a harder problem. Speak to your neighbours, these problems should be discussed.*

Euclidean spaces

1. Which of the following matrices A define an inner product B_A?

i) $A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 4 \end{bmatrix}$.

ii) $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

iii) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$.

iv) $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

For those matrices that do not define an inner product, explain why.

For those matrices for which B_A is an inner product determine

- the length of the standard basis vectors (with respect to B_A),
- a Euclidean isomorphism $f : (\mathbb{R}^n, B_A) \to \mathbb{E}^n$.

2. Show that the symmetric bilinear form

$$B : Mat_2(\mathbb{R}) \times Mat_2(\mathbb{R}) \to \mathbb{R} \; ; \; (X, Y) \mapsto \text{tr}(XY)$$

is not an inner product on $Mat_2(\mathbb{R})$ by determining $X \in Mat_2(\mathbb{R})$ such that $B(X, X) < 0$.

3. Is the symmetric bilinear form

$$B : Mat_2(\mathbb{R}) \times Mat_2(\mathbb{R}) \to \mathbb{R} \; ; \; (X, Y) \mapsto \text{tr}(X^t Y)$$

an inner product on $Mat_2(\mathbb{R})$?

4. Let V be a finite dimensional $\mathbb{R}$-vector space and $\mathcal{I}(V) = \{B \in \text{Bil}_{\mathbb{R}}(V) \mid B \text{ is an inner product}\}$. Is $\mathcal{I}(V) \subset \text{Bil}_{\mathbb{R}}(V)$ a subspace?

5. Show that a Euclidean morphism $f : V_1 \to V_2$, where $(V_1, \langle , \rangle_1), (V_2, \langle , \rangle_2)$ are Euclidean spaces, is length-preserving (so that $||f(v)||_2 = ||v||_1$).