Worksheet 6/25. Math 110, Summer 2012

*An asterisk * denotes a harder problem. Speak to your neighbours, these problems should be discussed.*

Linear morphisms

0. Show that $\mathbb{K}^n$ and $\mathbb{K}^{[n]}$ are isomorphic as $\mathbb{K}$-vector spaces. Recall that $[n] = \{1, \dots, n\}$. Therefore, you must define a function

$$f : \mathbb{K}^n \to \mathbb{K}^{[n]},$$

which is a bijective $\mathbb{K}$-linear moprhism.

Bases, dimension

1. Let V be a $\mathbb{K}$-vector space, $\mathcal{B} = (b_1, \dots, b_n) \subset V$ an ordered basis of V. Show that the $\mathcal{B}$-coordinate morphism defined in class is a bijective $\mathbb{K}$-linear morphism.

2. Let S be a finite set, $V = \mathbb{K}^S$. Show that the subset $\{e_s \mid s \in S\}$ is a basis of $\mathbb{K}^S$.

In particular, what is a basis of $Mat_{m,n}(\mathbb{K}) \overset{def}{=} \mathbb{K}^{[m]\times[n]}$?

3.* Explain why $\{e_i \mid i \in \mathbb{N}\} \subset \mathbb{Q}^{\mathbb{N}}$ is not a basis of $\mathbb{Q}^{\mathbb{N}}$ - your conclusions from a previous worksheet will help with this problem.

4. Which of the following subsets are bases:

$$\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\} \subset \mathbb{Q}^3,$$

$$\left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 100 \end{bmatrix}, \begin{bmatrix} -2 \\ 10 \end{bmatrix} \right\} \subset \mathbb{R}^2,$$

$$\{f, g, h\} \subset \mathbb{Q}^{\{1,2,3\}},$$

where

$(f(1) = 2, f(2) = 0, f(3) = 1), (g(1) = 0, g(2) = -1, g(3) = 1), (h(1) = 1, h(2) = 1, h(3) = 0).$