Worksheet 6/21. Math 110, Summer 2012

An asterisk * denotes a harder problem. Speak to your neighbours, these problems should be discussed.

Linear independence, span

1. Prove: if $E \subset V$ nonempty, V a $\mathbb{K}$-vector space and there exists $v \in E$ such that v is a linear combination of some of the other vectors in E, then E is linearly dependent.

2. Is the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ an element of $\mathrm{span}_{\mathbb{Q}} E$, where

$$E = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}?$$

If $\underline{b} \in \mathbb{K}^m$ and $E \subset \mathbb{K}^n$ is a (nonempty) collection of finitely many vectors, how can you determine if $\underline{b} \in \mathrm{span}_{\mathbb{K}} E$?

Linear morphisms

1. Check that $\mathrm{Hom}_{\mathbb{K}}(V, W)$ is a $\mathbb{K}$-vector space, ie, check that $f + g, \lambda f$ are linear and that $0_{\mathrm{Hom}_{\mathbb{K}}(V,W)}, -f$ are well-defined.

2. Let $f \in \mathrm{Hom}_{\mathbb{K}}(\mathbb{K}^n, \mathbb{K}^m)$ be a $\mathbb{K}$-linear morphism, A_f the standard matrix associated to f (defined in class and in the notes). Understand why

- f is injective if and only if A_f has a pivot in every column,

- f is surjective if and only if A_f has a pivot in every row.