Math 110, Summer 2012 Short Homework 1

Due Wednesday 6/20, 10.10am, in Etcheverry 3109. Late homework will not be accepted.

Some warm-up calculations

1. Row-reduce the following matrices to reduced echelon form:

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}.$$

2. For the following matrix equations determine whether a solution exists. If so, determine all possible solutions.

$$A\underline{x} = \underline{0}, \quad B\underline{x} = \underline{0}, \quad A\underline{x} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

3. Without using determinants show that A is invertible.

Problems from the notes

4. Show that $\mathbb{Q}$ is not a vector subspace of the $\mathbb{R}$-vector space $\mathbb{R}$ (where we have the 'usual' notions of addition and scalar multiplication).

5. Prove that, if (V, α, σ) is a $\mathbb{K}$-vector space, and $\lambda \in \mathbb{K}$, then $\sigma(\lambda, 0_V) = 0_V$. (*In your solution you can write* $\lambda \cdot 0_v$ *instead of* $\sigma(\lambda, 0_V)$)

6. Prove that V and $\{0_V\}$ are subspaces of a $\mathbb{K}$-vector space V.

7. Prove: if a nonempty subset $U \subset V$ satisfies Axiom SUB then U satisfies Axiom SUB1, SUB2 and SUB3.