# **Math 110, Summer 2012** Short Homework 10

*Due Monday 7/30, 10.10am, in Etcheverry 3109. Late homework will not be accepted.*

*0. Was this homework assignment too easy/too difficult/about right? Any other comments are welcome.*

**Calculations**

1. For the following symmetric matrices $A \in \mathrm{GL}_n(\mathbb{R})$ determine $P \in \mathrm{GL}_n(\mathbb{R})$ such that

$$P^t AP = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{bmatrix} \quad d_i \in \{1, -1\}.$$

i) $A = \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}$,

ii) $A = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$,

iii) $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

What is the signature of each of the corresponding bilinear forms $B_A \in \mathrm{Bil}_{\mathbb{R}}(\mathbb{R}^n)$.

2. For the matrix $A$ in *ii*) above, determine an ordered basis $\mathcal{B}$ of $\mathbb{C}^3$ such that

$$[B_A]_{\mathcal{B}} = I_3.$$

(*Hint: proceed as you would in the real case, except now you can use the fact that you are allowed to find square roots of negative numbers. See the proof of the classification of nondegenerate symmetric bilinear forms over $\mathbb{C}$ in Section 3.2.*)

**Proofs**

3. Prove: let $B \in \mathrm{Bil}_{\mathbb{K}}(V)$ be nondegenerate and symmetric, where $\mathbb{K}$ is ANY number field. Then, there exists an basis $\mathcal{B} \subset V$ such that

$$[B]_{\mathcal{B}} = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{bmatrix}, \ d_i \in \mathbb{K}.$$

(*This is a generalisation of the results of section 3.2. You just need to copy the proofs of the theorems in that section.*

Deduce that for any symmetric $A \in \mathrm{GL}_n(\mathbb{K})$ there is $P \in \mathrm{GL}_n(\mathbb{K})$ such that $P^t AP$ is diagonal.

4. Let $B \in \mathrm{Bil}_{\mathbb{K}}(V)$. Prove that $B$ can be written uniquely as $B = B_s + B_a$, where $B_s \in \mathrm{Bil}_{\mathbb{K}}(V)$ is symmetric and $B_a \in \mathrm{Bil}_{\mathbb{K}}(V)$ is antisymmetric.

(*Hint: you will need to use that $Mat_n(\mathbb{K}) = S_n \oplus A_n$ (recall SH4, Q3).*)