The University of Chelm

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When our son was 5, and we chose the first school for him, I brought to the school's principal the math curriculum from Singapore. It turned out that he considered another curriculum, *Investigations in Number, Data, and Space,* and I managed to borrow from him an examination set.

It was a puzzling bunch of teacher's materials showing vitually no connection with mathematics. It took only a few minutes to locate on the Internet a review, where *Investigations* earned an F in every aspect of evaluation. With somewhat greater effort, I found on the web a research paper authored by Anne Goodrow (in fact, the summary of her PhD thesis, posted at the website of TERC, the agency that developed *Investigations*) that claimed superiority of the curriculum. So, I sent to the principal my findngs where in particular explained why the superiority claim was a farce.

Our son, who is now in grade 5, likes listening to the storyteller Joel ben Izzy, especially to his stories about Chelm, the mythical jewish town of fools. In one of the stories, chelmites spend all their tresury to build a wall that would protect the town from rain. When this fails, and they find themselves amidst raining water and without money, they try to solve both problems by changing the meaning of the words. Suddenly they have plenty of money — but no water!

I recall this story because recently, in the Wikipedia article about Math Wars, I found a mention of that same research paper as the main evidence that *Investigations* produce superior results. Just as 6 years ago, it suffices to open the paper[1] to find out that it confirms nothing of the sort. I wonder if those who posted this paper to the TERC website as a piece of evidence, or those who quote it in Wikipedia as a scientific source, actually ever read it. For if they did, the only way how they could arrive at the requisite conclusion would be by using the strategy of the citizens of Chelm.

[1]Called *Modes of Teaching and Ways of Thinking*, and readily available at the TERC website: http://investigations.terc.edu/impact/impact-studies/modesofteaching.cfm

Let's open the paper. Three tasks: 12 two-digit additon problems, 12 two-digit subtraction problems, and one activity called "The Number of the Day", were suggested to each of the three groups of 10 second grade students taught in three different classroom environments labeled as "traditional," "transitional" (or "mixed"), and "constructivists." We are told that the latter two were using *Investigations*, but only the last group was regularly exposed to "constructivist" activities. We are told nothing about the curriculum used by the first group, extent of instruction received by the students, training of their teachers, or anything else that would allow one to gauge ones expectations.

Here are the conclusions of Anne Goodrow.

(i) There was no significant difference between the three groups of students in solving addition problems and subtraction problems without regrouping.

(ii) In subtraction problems requiring regrouping students taught in the "constructivist" environment performed better, as illustrated by Figure 5.

(iii) In the "Number of the Day" task children from "constructivist" and "mixed" groups performed better, which is illustrated by Figures 1 and 2.

Here are my consclusions:

(i) and (ii) rather characterize academic integrity of the author, while Figures 1 and 2 warrant a conclusion opposite to (iii).

Let us begin with (i) and (ii). In the time when thousands of scientists and engineers are putting up a giant hadron collider in Europe to find out what our Universe was like a few milliseconds after the Big Bang, their colleagues from departments of education cannot put two and two together to realize that in their study of the constructivist classroom, the rest of the World is their control group.

We all come from traditional classroom, and we know what we learn, and when, and how well, and it is obvious to those of us who live outside the city wall of Chelm that *that* group of 10 "traditionally" taught second graders in Anne Goodrow's study had *lousy* curriculum, or *unskilled* teacher, or *inadequate* training, and most likely all of the above.

For, in a normal second grade, students are trained to perfom the standard algorithms with 3-digit numbers and with fewer errors, taught to understand and be able to explain why the algorithms work, and are also taught to do calculations within 100 without paper or pencil.

I open my son's old textbooks and workbooks from Singapore and find that mental addition and subtraction within 40 are taught in Kindergarten, within 100 in grade 1, that the first half of grade 2 includes detailed and conceptual explanations of the addition and subtraction algorithms, with and without regrouping, and for whole numbers within 1000. In the dozens of practice exercises solved by my son I hardly find any errors at all.

The less-then-75% of correct answers to 2-digit subtraction problems shown on Figure 5 of Anne Goodrow's paper indicate poor training. Furthermore, Figure 6 shows that student R was taugh *incorrectly*. Namely, the student was apparently encouraged to record some steps which should be performed mentally (since there is no adequate space for them on paper — look at the "18" written on the side and read the student's explanation), and was not warned to avoid recording the tens remaining after regrouping in the hundreds position.

At the same time, performance of the constructivist group is assessed incorrectly. The exercise $28 - 9 = 19$ is not much different from $18 - 9 = 9$ which certainly falls into the range of what is *remembered* by all students. No knowledge of negative numbers was needed here, nor was such knowledge demonstrated, as a matter of fact, by student N (see Figure 9). Namely, $9 - 8 = 1$, not -1 as N claims, $-1 - 21 = -22$ and is *not* the same as $20 - 1 = 19$. Likewise, the lengthy ad-hoc explanation of student C (see Figure 4) of a procedure which actually amounts to execution of the standard algorithm demonstrates *poor* skills of ad-hoc calculation. Namely, since 29 is one less than 30, $34+29$ can be easily found to be one less than $34+30 = 64$.

Thus, we should conclude that performace of *all* the three groups in basic tasks of subtraction within 100 was on the low side compared to what is normally expected from second graders exposed to traditional training — in the normal meaning of the words. It is easy to believe that in a regular US classroom students receive poor mathematics instruction or no instruction at all, but such classrooms are not a valid reference point, for it is only in the classrooms of Chelm *traditional math education* means a lack thereof.

Furthermore, conclusion (i) is misleading: There is an obvious and substantial difference between constructivist and traditional training. There is a mathematical reason why the standard addition and subtraction algorithms are performed "from right to left." Those who are not taught to understand it, face challenging complications when they try to operate with several-digit numbers. Anne Goodrow chose the task of addition of 2-digit numbers, where this distinction does not surface yet. Had she chosen 3-digit additoon and subtraction problems, like $647+278$ or $647-278$, the students from her constructivist classroom who were not taught to start with ones and proceed to tens, would have to perform more operations making therefore more errors than those subject to adequate traditional training. Choosing a task where the truth does not have a chance to surface does not exactly fit the notion of objective study — that is, unless the meaning of the words is reversed.

Let's turn now to "The Number of the Day." In the mathematical language, the task given to the students can be formalized as the following problem: *Prove that there exists a whole number that can be represented in at least five different ways using arithmetic operations.* The following proof shows that the problem is trivial: $0 = 0 - 0 = 0 + 0 = 0 + 0 + 0 = 0 \times 0 = 0 \times 0 \times 0$.[2] It does not require much imagination, and is not related to any interesting or important mathematical theorem.

This is actualy a very basic and important point in teaching mathematics. If you cannot explain what exactly is the non-trivial mathematical theory behind an exercise you assigned to your students, then you didn't teach them any mathematics at all. That is, as long as the words "mathematics" and "teaching" mean what they usually mean, and not what they might mean in the town of Chelm.

Thus, here how it goes: When you explain to me what is the non-trivial mathematical theorem behind the task "The Number of the Day," then I will say that Professor of Mathematics of the University California Berkeley Alexander Givental is a fool who knows nothing about mathematics. But until then I will maintain that Anne Goodrow, together with her colleagues from TERC who endorse her study, and together with the inventors of the activity "The Number of the Day" are among the Honorary Faculty of the Universiy of Chelm.

Let us, however, examine Figure 1. What a mathematician can see in it, is that the student attempted to interpret the silly, non-sensical task in a meaningful way. Namely, he set himsef out to *find a positive integer that can be represented as the sum of two positive integers in at least five different ways*. He even estimated the *smallest* of such integers; that's why he uses 12 (the smallest one is actually 10).

In fact, as Anne Goodrow admits, *most* students in the "traditional" group interpreted the silly problem the same, meaningful way. Thus they certainly *outperformed* not only the other two groups, but also the authors of

[2]Note that example $59 = 59 - 59 + 59$, mentioned in the paper as a valid response, justifies our assumption that these 5 representations of 0 are considered by the authors of the task as different ones.

the task, by demonstarating better mathematical taste and common sense. Yet, they received no recognition for this. Moreover, we read: "Children in the constructivist group created significantly more correct number sentences than those in the traditional group," and this fact is readily illustrated by Figure 1: The student made an error in the last of the five representations.

This is the moment to appreciate the true value of good mathematics education! What if this student and his team-mates got all their 5×10 representations right? Would Anne Goodrow admit that traditional math education (that is, simply *math education*) is superior? Would TERC then abandon its ridiculous curriculum? Many student's souls and millions of dollars could be spared this way! Well, let's not fool ourselves: At the University of Chelm, many words — true, false, science, research, imagination — have long lost their meaning.

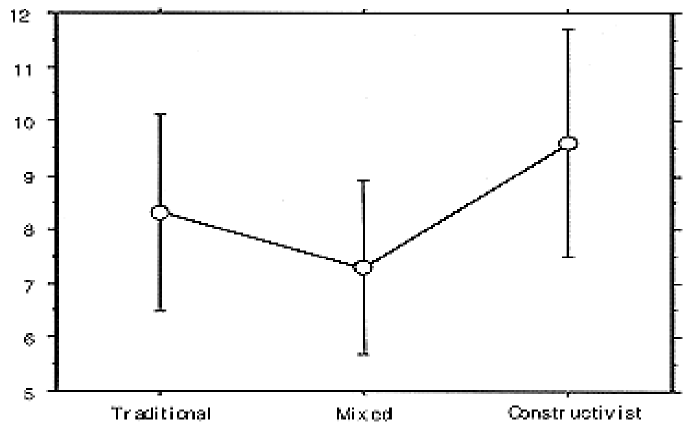

Figure 5. Mean number of total correct responses by type of instructional group.

Subtraction Strategies

R.'s work:

$$\begin{array}{r} 1\cancel{2}\cancel{8}18 \\ -\ \ 9 \\ \hline 19 \end{array}$$

I: How did you think about that one?
R: Cause you can't take nine from eight, so you cross it out and get um, you cross it out, you cross the two out and make a 1, and take that one and put it with the 8, and the 8 has 2 numbers, and then 9 plus 9 equals 18.

Figure 6. The standard subtraction algorithm in the Traditional group.

N.'s work:

$$\begin{array}{r} 28 \\ -\ 9 \\ \hline 19 \end{array}$$

N: 28 minus 9.
I: Now this one's tricky.
N: No, it's not. 19.
I: How come it is 19? Tell me what you think?
N: Wait. um
I: That's right. Why is it right?
N: Because 9 minus 8 is -1.
I: 9 minus 8
N: No, I don't want to start with that. Let's see. Hmmm. I have to start with that. 9 minus 8 equals negative 1 and negative 1 minus 20. That's 20 minus 1. Equals 19.

Figure 9. Negative numbers in the Constructivist group.

$$\begin{array}{r} 34 \\ +29 \\ \hline 63 \end{array}$$

C: This kind of problem I can't do the same [in the same way as the previous problem].
I: You can write as much as you want. If you want more paper I have more too.
C: No thank you, I'm just —the 30, and then I always add the 29. I take 20 from the 9, and here's the 30 and the 4. I'll just make a number string out of it, like this. 30. Cause I have all these numbers, I broke these two into smaller, which are easier to handle. 30 + 4 + 9 + 20, so first I only add the 9 and 4, but I could break up this 4 if I wanted.
I: If you wanted to you could.
C: I know that would equal 13, because I only need 1 more to make 10. If I take that 1 it would only be 3.
I: Umhmm
C: So here's the 1, here's the 3. I used this 1.
I: I understand.
C: and then it would be, so I add this 13. And then I add this and this which would be 50 because 3 plus 2 equals 5. I just have to add these two together, which would be 63.

Figure 4. Recomposing and adding tens as ones in the Constructivist group.

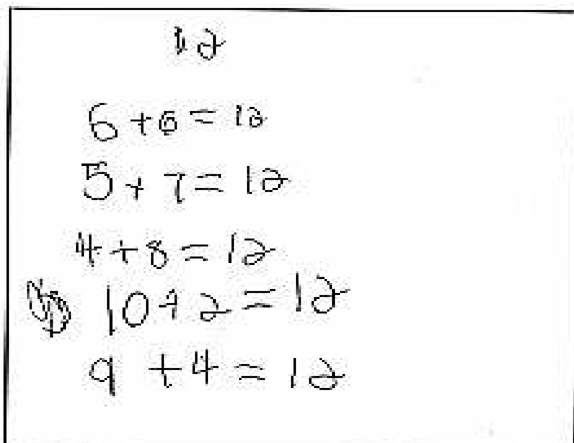

Figure 1. Example of the Number of the Day from Traditional group

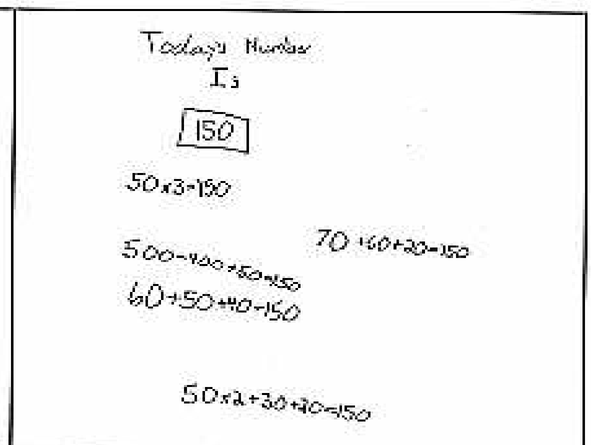

Figure 2. Example of the Number of the Day from Constructivist group