Introducing Singapore Math Program

by Alexander Givental

Usually a choice of educational programs and textbooks is a compromise between content, clarity, convenience, fun, cost, etc. It is actually a miracle that with Singapore math it isn't. In every relevant aspect the program "Primary Math" (shortly — PM) seems far better than anything else available in the US. Obviously, when the devil was busy spoiling American math education, he didn't realize that Singapore people speak English.

What is the secret of PM's success?

PM draws its inspiration not from fancy educational doctrines but from two natural resources — the subject itself and teacher's experience.

As opposed to everyday life, genuine mathematics is simple. It captures only very few very basic properties of the real world and shows how to manage them very efficiently. This simplicity and other features of mathematics are manifest in the program in multiple ways. Here are some of them:

- Small size. A year-long PM program consists of two textbooks and two workbooks of about 100 page each printed sparsely in large fonts and with lots of pictures. In symbol count, the total size can be estimated as equivalent to 40 (2nd grade) to 100 (5th grade) pages of "adult" reading material per year.
- The textbooks are unambiguous about what is to be learned in each topic. The lists of contents provide a clear "scope and sequence chart" for the whole curriculum.

- The ordering of sections neatly reproduces intrinsic logical relationships between the topics studied in these sections. Thus the very structure of the program reflects and teaches to understand — the layered nature of mathematical knowledge.
- In basic math, each topic is prerequisite to moving on to the next layer. In PM, this is efficiently used to minimize the need for bare drill: each new skill is mastered in the process of studying sequel topics.

The PM materials are marked with exceptional quality.

How is this achieved? All authors know the pitiful truth: good intentions are not enough to write a great textbook. The excellence of PM is a result of thoughtful editing *plus* decades of teaching practice which allowed the authors to understand many fine points in elementary mathematics and the ways they are best taught to kids.

For instance, it is a great idea to start each topic with pictorial explanations, but it takes a lot of practice to make them all as transparent as they are in PM. A result — textbooks truly designed for reading by kids themselves.

Another impressive feature is the amount of both fun and challenge sprinkled into textbooks and workbooks.

Some other advantages of PM:

• The very existence of textbooks written for children is important. Firstly, it teaches kids brain-on reading skills. Secondly, it is an efficient instrument in engaging parents into the education process. Thirdly, it provides an opportunity for independent study.

- The PM program should be very easy to teach. It usually suffices to follow the books literally without any omissions or supplements. The textbooks are self-explanatory; yet the program comes with Teacher Guides which are about 100 pages-per-semester long. As it is seen from samples, the Teacher Guides outline daily study plans, provide verbal descriptions of the pictorial explanations given in the textbooks, suggest in-class activities, supply answer keys and some extra problems.
- In addition to the US edition of PM, there exist: a book of supplementary drill exercises, a supply of manipulatives, several Singapore clones of the curriculum, some enrichment materials and interactive CD-ROM variations of the program.
- Professional development based on Singapore Math: The book *Elementary Mathematics for Teachers* by T. H. Parker and S. J. Baldridge uses PM as a model. There is a course at the Oregon State University (Eugene) taught to pre-service teachers which is based on this book. Reading this book is highly recommended to all elementary school teachers.
- The program is well-tested: in Singapore, in Israel, in the US in some private schools and public school districts, by homeschoolers. All reports indicate success and dramatic improvements.

Here are some words about PM from "experts": mathematicians involved with teacher's professional development programs, teachers using PM and other enthusiasts.

• From *Preface to Teacher's Guide for PM*, by Madge Goldman, President of Gabriala and Paul Rosenbaum Foundation:

Why is a private, non-governmental U.S. organization like the Rosenbaum Foundation creating American Teacher's Guides for a foreign country's proprietary mathematics books? Because the Foundation believes that Singapore's Primary Mathematics books are the best elementary school math books available in English ...

... Working with the professional mathematics community led the Rosenbaum Foundation to focus, not on reform experimentation, but on identifying those teaching materials and pedagogical practices that already have a solid, proven record of success. The mathematicians determined that the school math of Japan, China, Singapore, Korea, the former Soviet Union and a number of the small East European countries are all excellent. Further, Singapore's existing English language school math materials were found as good as their students' first place TIMSS standing would suggest.

The central idea of all of mathematics is to discover how it is that knowing some few things will, via reasoning, permit us to know much else - without having to commit the new information to memory as separate facts. Mathematics is economy of information, not its unnecessary proliferation. Basic mathematics properly presented conveys this lesson. It is the connections, the reasoned, logical connections, that make mathematics manageable. Understanding the structure of mathematics is the key to success. Everyone can be "good at mathematics", and this series, as has been proved in Singapore, shows how ...

... These deceptively thin texts were created with an impressive understanding of how children actually learn.

• From e-mail by Yoram Sagher, Math Deapartment Chair at Florida Atlantic University:

Singapore "Early Bird" (for Pre-K and K) and Primary Math" (for 1-6) are infinitely better than the University of Chicago series. ... I

have a record of the achievements of students in the Ingenuity project in Baltimore where I worked as a consultant for a little more than a year when they adopted the books. The results were indeed dramatic.

• From e-mails by Richard Askey, member of the National Academy of Sciences, professor-emeritus at University of Wisconsin–Madison:

The one private school I know well which is using "Primary Math" is Madison Country Day School just outside of Madison. They were probably the first school in the US to start using this series.

... The [public] school district is Marshall ... Marshall's scores on the Wisconsin state test have gone up a fair amount ...

... Good luck on convincing your school to use these books. They seem to be the best there is in English.

• Ray Tenebruso, math teacher from Madison Country Day School using Singapore Math:

Our students are doing quite well with it ... Moreover, they tend to understand what they are doing, rather than merely carrying out routines ... You probably have noticed how much mathematics is both done and foreshadowed in the texts ... that beginning algebra students frequently find troublesome - no technical language, no fanfare.

A topics is typically developed by beginning with what the students either already know or find intuitive. A topic then progresses by degrees, eventually becoming fairly sophisticated for the grade level. But, unlike other curricula with which I am familiar, the student gets a lot of support from the teacher, the text, and the well chosen workbook problems. Often the children do have the sense of discovery, but unlike some other curricula, the students are lead to exactly the discoveries we wish them to make.

Our students like mathematics, and that goes for the students who struggle with it, too. Students typically are confident in their approach.

If you have looked through the workbooks and problems sets, you have probably noted that there is not a lot of repetition. In a workbook exercise, almost each problem is a little different than the rest. There are times when some students simply do need more repetition to master a skill. At such times, it is not a big deal to make up some problem sets. Additionally, there are available a few books of extra problems from which a teacher could copy.

The workbooks are also good in a practical way, they solve a kid's problem of being organized and keeping track of homework. They also become a collection of worked problems that a student can look back to for later review.

Word problems are common. They usually play a part in teaching a topic, as opposed to merely some "applications" stuck on at the end of a topic. The word problems are also clearly and concisely stated and well defined. Again, a contrast to some other programs. It is the mathematics, the concepts and ideas that are stressed, not literary interpretation.

... It is actually hard to imagine what people could want besides the books. Anyone who understands the mathematics they are teaching should be able to do pretty well with the books.

I will say that Scott Baldridge, a professor of mathematics (not education stuff, I mean mathematics) can give teachers wonderful insight into the mathematics they teach . . . and he uses the Singapore books as an object of study.

• From the textbook *Elementary Mathematics for Teachers* by T. Parker and S. Baldridge:

The aim of this course is to develop an understanding of elementary mathematics at the level needed for teaching. The best way to do that is to study actual elementary school textbooks and to do many, many actual elementary school mathematics problems. The Primary Mathematics books were chosen for that purpose.

We will read and study these books with two goals in mind: understanding the mathematics and understanding the curriculum development. The Primary Mathematics books give an extraordinarily clear presentation of what elementary mathematics is and how it is organized and developed. They lay out the subject in depth, and they include a rich supply of exercises and word problems. The mathematics is always clean and correct, and topics are repeatedly covered from different approaches. Viewed from a broader perspective, these books provide much useful guidance about curriculum issues. They exhibit the principles of a well-designed curriculum better, it seems, than any textbook series currently available in English.

It is not surprising, then, that the Primary Mathematics books are also successful with children! The Third International Mathematics and Science Study (TIMSS) rated Singapore's elementary students the best in the world in mathematics (it also found that the curriculum is highly coherent). These beautifully designed books are a major factor in student success.

• From e-mail by Ron Aharoni, professor of math, Technion, Israel:

... there is a lot of material that we have written about these books they are really unique in the mathematical understanding implemented in them ... We are now working in 54 schools, very successfully. Next year it is probably going to grow to about 100 ... Will be glad to send you any material you wish in Hebrew (we are now translating a booklet I wrote into English). I also have a book, translated into English, about elementary math'l education ...

• From The Israeli Foundation for Math Achievements for All:

The Israeli Foundation for Math Achievement for All (IFMA) was founded on 2002 by teachers, mathematicians, high-tech industry people, and educators, with the mission of restoring the excellence math education ...

As a first project, the foundation took upon itself to introduce into Israel elementary schools the "Basic Math" books. This is a translation from English of the books that have been in use in Singapore at the period when Singapore consistently won the first place in the International Math Tests (TIMMS). "Basic Math" books are held in high esteem in many countries. Their strength is universal and is not culture-dependent. Mathematical principles are systematically built upon each other, in a consistent and mathematically correct order with a gradual transition from the concrete to the abstract ...

The "Basic Math" is in its second year as "field test" ... The results so far are very encouraging.

Here are some news reports from official organizations about Singapore Math:

• From Press Release, 9.10.2002 First US-Singapore Government to Government MOU on Education signed on 9 Sep.

... 3. As a start, Singapore and the US will be working together to study the best approaches to the teaching and learning of science and mathematics. There will be a joint US-Singapore study of Singapore's approach in the teaching and learning of Mathematics. Singapore students had performed well at the Third International Mathematics and Science Study (TIMSS-95) and its follow-up study in 1999, coming in top among 38 nations in both studies. One of the aims of this study is to assess the effectiveness of the use of Singapore mathematics textbooks in US schools. The joint study will also enable Singapore to gain a better understanding of the effectiveness of the strategies used in teaching mathematics in our schools.

• 11.03.2003 Singapore model led to high marks at four schools by Erik Kelderman (Staff Writer)¹:

A school system study, released quietly in February, showed that students in a small mathematics pilot program did significantly better than their peers who were not in the program. "... For every assessment, at every grade level, students in the Singapore Math pilot schools performed significantly higher" than schools that did not have the program, the report states. The Singapore Math schools also scored higher, generally, on the CTBS mathematics and mathematics computation tests, the report states. And the two schools that implemented the Singapore program more fully scored higher in both areas than the other two schools using the curriculum, according to the study.

These days, the army of homeschooling parents is searching actively for the programs that work best for their kids. Their reviews provide the most dynamic evaluation of existing programs. Here is how a reviewer ² of homeschooling math programs describes the essence of PM:

 $^{^{1}\}rm http://www.gazette.net/200344/montgomerycty/education/184665-1.html <math display="inline">^{2}\rm Nicki$ Bradley, http://www.suite101.com/article.cfm/18208/106605

The method by which the Primary Mathematics teaches math is the one that I think really works to create math-loving children. It guides children through the *whys* of math while teaching the *hows*. This program creates mathematical thinkers and problem solvers! Children who learn well with Singapore math will come away really understanding math on a level that many of us missed out on as children. Instead of teaching one rote method by which to solve all similar problems, Singapore Math teaches several different ways to look at the problem and really understand it. Mental Math is taught from the very beginning and becomes second nature throughout the program. Story problems are no longer something to fear because math makes sense! ... Bottom line: if you are afraid to teach your child math and did not do well yourself, this program will teach you too! It introduces math in a way that isn't traditional in the US and results in children who really think and understand math without any of the drill work or repetition that turns so many of us off to math.

Here are some highlights from reviews by homes chooling parents: 3

- Singapore Math is so well laid out, the illustrations are brilliantly put together, the lessons are so simple to follow through, the cd-rom is very entertaining, and although the teacher guides are very well written... I just have not needed to use them... My favorite part ... is how they bring out an advanced way of learning and make everything so "simple".
- I did quite a bit of math in college and have an electronics degree. I find it teaches the process of thinking about math just the way I do it in my head. It is so easy to teach. My oldest who is not mathematically inclined has caught up and passed the place she ought to be with her peers, and my 2nd, a math genius like me :-), loves to do lessons and begs to do more.
- It is easy to understand, well laid out, very very visual and explains concepts in tiny bite size portions so that even I grasp basic concepts at a more solid level.

 $^{^{3}}$ http://homeschoolreviews.com/reviews/singapore.html

- ... the word problems are outstanding! Students are required to really think through the problem instead of just plugging in rote figures. ... The child who needed help memorizing, turned out to be quite adept at setting up these word problems!
- Singapore math makes doing math so much more fun and easy than when I was in school. I can't believe how much I've learned that I missed all those years... Singapore is very heavy in emphasizing the mental way of doing math instead of counting on fingers or writing every little thing down ... I also like the way that Singapore almost never shows only one way to do a problem but shows two or more ways to do it or think about it. ... What Singapore does do well is prepare children to apply what they've learned to other, more complicated math problems. Something that they need in order to do well with higher level math, not just rote memorization.
- There's not endless repetition of each topic ... and there are lots of story problems ... They use a lot of pictures and games to make the extra drill on new things ... fun. In general, Singapore math goes more in depth than comparable American programs.

One more thing needs to be said about PM. I have never heard of anyone falling in love with math over a US curriculum. Here are some quotations from homeschoolers using PM:

- WE LOVE THIS PROGRAM ...
- If there is any math cd-rom you buy, please, please, let it be it ...
- She really likes how she can wizz through some of the chapters, by herself!!!
- This is OUTSTANDING! ... She begs for her math...when she once HATED it so much.
- It's soooo wonderful! ... The girls love it, they are coaxed through phonics (younger) or grammar (elder) by the promise that their MATH! is waiting for them.
- Awesome, amazing program! ... I am math-phobic as they come, but Singapore math has me as excited as my girls.