

## Practice Exam

**1.** Derive Kepler's 3rd law: The squares of periods of revolution of planets around the Sun are proportional to the cubes of the major semiaxes of their elliptic orbits.

**2.** Give an example of a function  $f$  with  $f_{xy} \neq f_{yx}$ .

**3.** Find all differentiable functions  $f(x, y)$  such that  $f_x = y \exp xy$ .

**4.** Compute the gradient of the function  $1/|\vec{r}|$ .

**5.** Find all values and partial derivatives at the point  $(a, b) = (5, 4)$  of the function  $x(a, b)$  defined implicitly by the equation  $x^4 - ax^2 + b = 0$ .

**6.** Find and classify all critical points of the function  $f = xy - x^3/3 - y^3/3$ .

**7.** Find the global maximum and minimum values of the function  $\sqrt{x^2 + y^2}$  in the region

$$-1 \leq x \leq 3, \quad -2 \leq y \leq 4.$$

**8.** Find all critical points and critical values of the function  $x + y + z$  under the constraint  $2xy^2z^3 = 1$ .