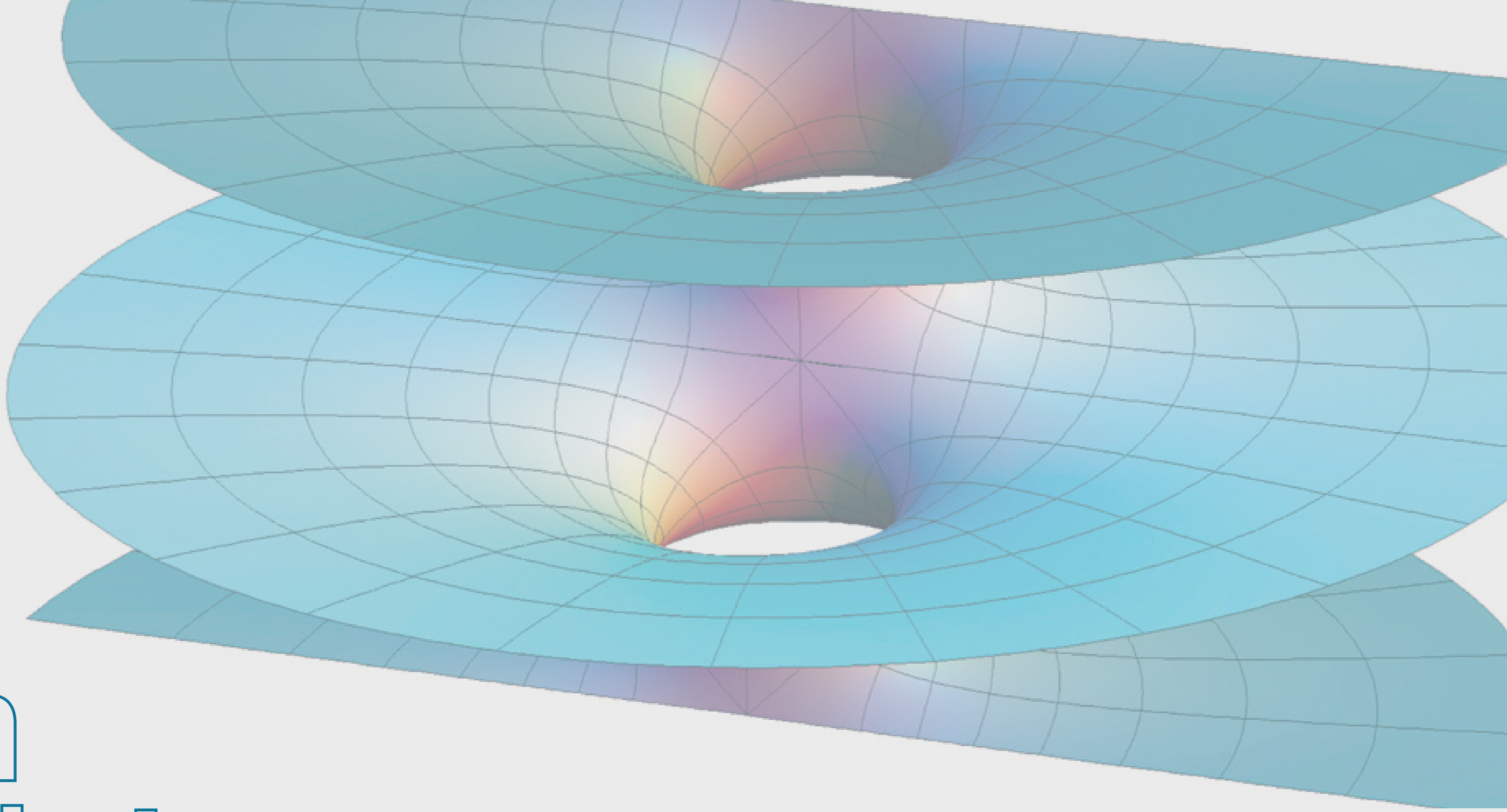
Affection for Certainty

EDWARD FRENKEL *in conversation with Matthew Putman*



Above: Reiman surface—an attempt to visualize a one-dimensional folded plane, a building block for understanding multiple dimensions. Illustration credit: Dick Palais, 2014.

Edward Frenkel, professor of mathematics at California Berkley, authored *Love and Math*, part biography part attempt to explain, among other things, his research in the field of a unified theory of mathematics (sometimes called the Langlands program). He sat down with physicist, Matthew Putman, to talk about the relationship between physics and math, love, and the shape of the universe.

MATTHEW PUTMAN So, you're getting famous for being a mathematician, but you still have a grounding in physics. Does math have a beauty in its own right, even if it doesn't have a physical representation? And is that attractive to you?

EDWARD FRENKEL Yes, it does. And in fact, this is one of the main themes in my book, [*Love and Math: the Heart of Hidden Realilty*] that mathematics is—in my view—separate from the physical world, and the mental world. They are connected, very deeply connected. But there are many mathematical theories. Some mathematical theories manifest themselves in a physical reality. But some of them don't. And maybe they will, at some point. We don't know. But there is an inner logic of mathematics that moves us to ask, and try to answer some deep questions. Einstein's General Relativity Theory was based on the work of mathematician Bernhard Riemann, which was done 50 years earlier, about curved shapes, and what it means to have curved space, which is not embedded in any other. He was the first one to tackle this. There were others, like Gauss, who also had ideas about this, but Riemann was the first to have a systematic theory. At that time it was like, why would anyone bother? Because of course everybody "knew" quote-unquote that our world was flat. Of course, it turns out it's not flat, that actually our space is curved, because, for example, a ray of light bends near a star.

MP Right.

EF But then, if it's curved, then where is it embedded? That's how our brain is wired, because we only imagine the space we inhabit as a flat space. So, anything that's curved, like this glass, always lives inside a flat landscape. It's very difficult to grasp the idea that a space, which is curved, could exist by itself—that a sphere could exist by itself, without being embedded in three-dimensional space.

MP Right. Which is the idea of the singularity in the first place.

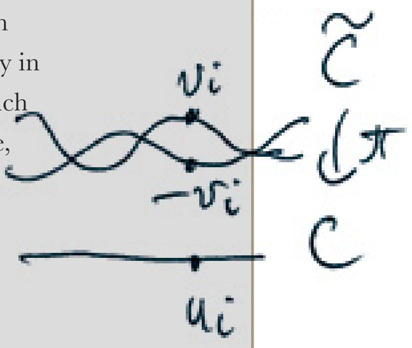
EF That's right.

MP There is no space outside of the singularity, at time zero.

EF That's right. That's a good example of this sort of subtle interplay between math and physics—that mathematicians ask these questions way before philosophers, way before physicists.

MP Right. But see, this is also the danger. I work in nanotechnology—so I deal with matrices. Almost everything that I deal with is in discrete linear algebra terms. I rarely deal with continuous systems at all. There's hardly ever a confusion, in my mind, of when something is abstraction for the sake of being a tool, and when something is abstract in a more pure, abstract way. But when you're dealing with quantum field theory it seems to me that you always have to keep yourself in check, and to figure out, "Am I dealing with something that has a physical representation? Or is this just beautiful math?" I think certain physicists step out of the realm of physics, without admitting that they've stepped out. Do you think that happens?

EF Absolutely. One has to keep track. It's almost like I have to know which hat I am wearing, the hat of a mathematician, or the hat of a physicist. If I am a mathematician, I'm interested in all possible mathematical theories. A mathematical theory can be consistent, and sound, without having anything to do with physical reality. But if I'm a physicist, and I'm interested only in the things that describe the universe the ultimate judge of such a theory is an experiment. I like to be a mathematician more, in a sense, because it gives me a little more freedom.



MP Yeah. You're not constrained by reality. [*laughs*]

EF That's right. And in fact—

MP Although, it is a reality in itself, of course.

EF Exactly. It is a separate reality.

MP Once it's created, whether in the imagination, in the mind, or especially within the framework and the constraints of math itself, and theorems and so on...

EF That's right. The mathematician, Georg Cantor was the first to realize that there are different kinds of infinity. Other mathematicians were puzzled by this, and frightened by this, and said he couldn't do it. And he said, "Yes, I can. The essence of mathematics," he said, "lies in its freedom." There are no boundaries. Within it, of course, there are rigid rules of logic. But you cannot constrain yourself. You can go as far as you can. And that's the beauty of mathematics, I think.

MP I don't even know if you believe this or not, because I'm not sure about how I feel about this—but would you say that a great mathematician can therefore make a great physicist, because they are not constrained by their imagination? Once you have a mathematical concept that makes sense within the limitations of physics, you can say, "Great. Now that moves into the realm of physics." If it never hits that, it's still beautiful math.

EF Right. But the difference is in the focus. So, the focus of a mathematician is to have a consistent theory, and to go as deep as possible. But the focus of a physicist is to describe the universe. So, one cannot replace the other. There is this tension, you know? I'm far from saying that it's mathematicians who really are going to discover the best physical theories. You have to have that focus and motivation, and you have to have your eye on the ball all the time, if you're a physicist. And if you're a mathematician, it's almost like you have many different love interests—you're not monogamous. It's almost like a polygamy of knowledge. In the great landscape of theories you love all of them equally. But if you're a physicist, you love the one that describes your universe, you know? And I respect that.

MP Right. And, then we go right back into that trap of what becomes beautiful, but maybe not real.

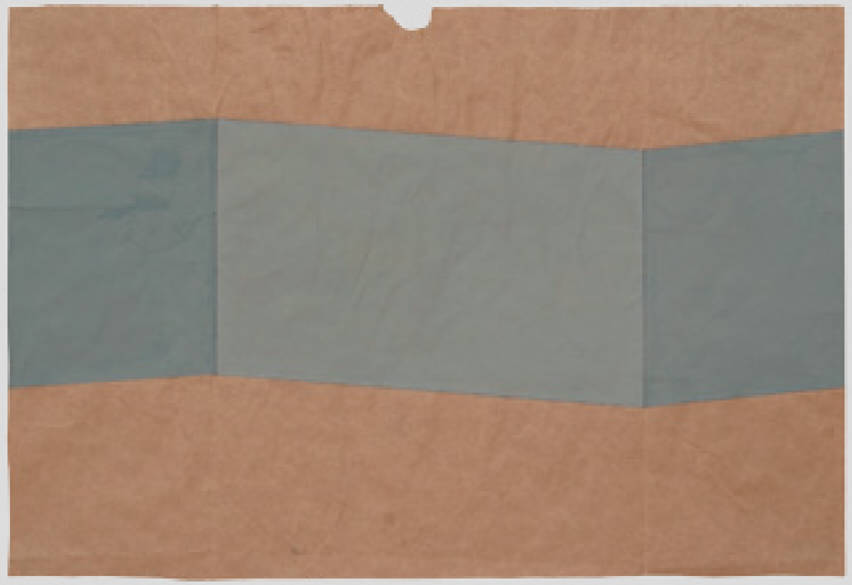
EF That's right.

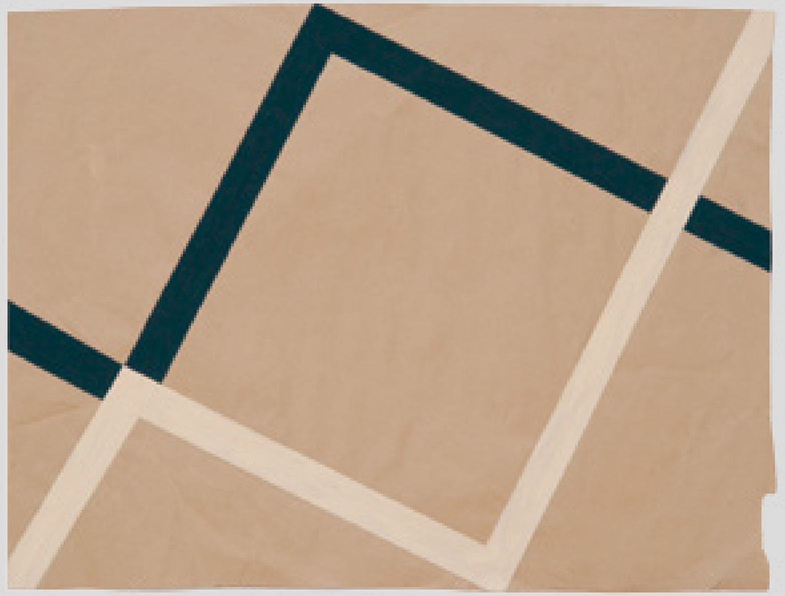
MP Things become sort of a pet loved theory, that's not really a theorem yet. Most physics grad students that I've spoken to, you ask them what they think of the multiverse and most will say, "Yes, I believe there is a multiverse." To me, the fact that you're saying, you "believe" in a multiverse is insane.

EF Yeah, because it's not the function of science to believe.

MP You don't believe in anything. And yet now we've gotten to

Above: Ben Berlow, *Untitled*, 2013. Acrylic and gouache on paper. Courtesy of Rawson Projects, New York.





a point where things have become so precious, because of maybe a mathematical elegance—like string theory. Perhaps it's right. Perhaps it's not right. But it becomes so precious that you start to accept it, almost as a religious acceptance. And I'm finding that very surprising.

EF That's right. I have to be aware—self-aware—that I am a mathematician, not a physicist, right? So, if I start talking about that theory being the theory of the universe, I have to be very careful. Has it been experimentally confirmed? What are the ways to confirm it? What are the ways to experimentally distinguish my theory from someone else's? What are the ways to falsify my theory? Right? Which is ultimately the test that a physical theory has to pass. And multiverse is a good example of this changing category, because string theory is a beautiful mathematical idea. It may well be the theory of the universe, or maybe part of the ultimate theory of the universe. We don't know.

MP Or maybe not at all.

EF But it has a fundamental issue, which is that string theory—or more properly, superstring theory—can only be consistent mathematically in 10 dimensions, in 10 space-time dimensions. We only observe four space-time dimensions—three spatial dimensions, and one time. So, what happens with the remaining six dimensions? In principle, it's possible that our world is 10-dimensional. For example, if you have a tube of a very small radius, it might appear to you as a line. The extra circle could be so small, that it's almost invisible.

MP Right on nanotubes when I'm visualizing it from above it doesn't look like a three-dimensional object at all.

EF That's right. So, the dimensionality of our space-time is a big question. It could well be that there are extra dimensions, but they are wrapped on something very small. It could be that there is one extra dimension, and that extra dimension is wrapped on a circle with a very small radius, which we cannot see. If there was just one extra dimension to accommodate, there would only be one choice—that is a circle—because a one-dimensional object could either extend infinitely far, or if it's finite, it has to be a circle. It has to run up on itself, so the only parameter you would have is the radius of that circle. And that's it. Now, if there were two extra dimensions to accommodate, there would already be more choices, because it could be a sphere, or it could be the surface of a donut. Or it could be the surface of a pretzel, and so on—what mathematicians call Riemann surfaces. There are already more choices, more possibilities. But now, imagine we actually have to accommodate six extra dimensions, because that's the only way to have a consistent string theory. So there has to be a six-dimensional shape that we don't see, which is very small. But what is it? It turns out that there are 10 to the 500 choices, by some estimates—just an unimaginably large number. And one of the biggest questions of string theory is, which one is it? The reason why this idea of multiverse became so popular is because some physicists said, "Actually, we don't know which one. All of them are realized. Each of them gives rise to its own universe." Depending on which shape appears, and which shape those six extra dimensions are wrapped on, you will have different universes, with different laws of nature. And maybe only one of them will support a conscious being who will ask the question, "Why are we here?" So, it's kind of like a marriage of the antropic principle and string theory.

It's an interesting question. However, my comment is that we're actually so far away from knowing what string theory really is. We have only very rudimentary understanding. It's not really a theory. It's a bunch of tricks. So my feeling is that we should put more effort into actually trying to figure out what string theory is, before saying that this is the only alternative. Maybe if we work harder, in 10 years, in 20 years, we will find some new principles,

Opposite (top): Ben Berlow, *Untitled*, 2013. Graphite and gouache on paper. 31.5 x 25 in. (bottom) *Untitled*, 2013. Graphite and gouache on paper. 22 x 32.5 in. Courtesy of Rawson Projects, New York.

$$0 \to E_1 \to E \to E_1' \to 0. \quad \phi : E \to E \otimes K$$

some new ideas that will say, "Actually, this six-dimensional space is such-and-such," so you don't have to say that all of them are realized randomly.

MP Yeah. I think you went very quickly by something that is important here—that we're several layers deep into something that's not falsifiable.

EF That's right.

MP So, a discovery is only a part of a process of discovering the universe, and there is no end-point to this. Say somebody creates a painting—that painting is completed. It's on the museum wall. There are things in life that get completed. Science doesn't.

EF But once you choose your axioms, then any mathematical theory is like a work of art, that is completed. It is there, it is true, it is valid, right? Unlike a physical theory where you would actually need to have an experiment. And even then, you might have an experiment but there are still some areas in which your theory doesn't apply. In physics, you can spend all of your life on this particular theory, only to find out that experiment proves it wrong, or there is no experiment.

MP It's interesting because as humans, how does that actually feel, when you are the father of a physical hypothesis, which has been shown not be true, after a 30, 40-year career?

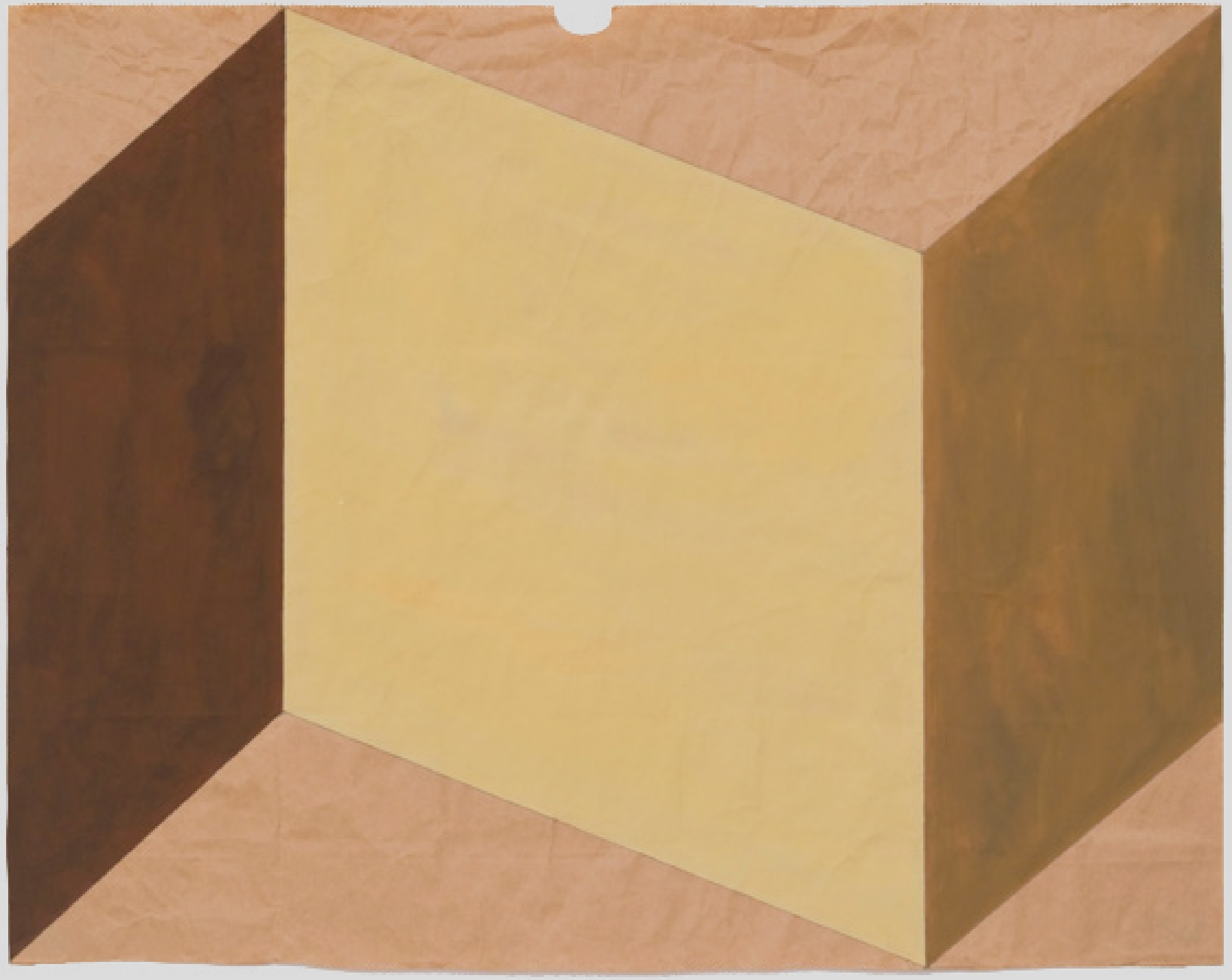
EF I mean, of course mathematicians also get to be disappointed and frustrated. We find mistakes in our proofs, a famous example being Andrew Wiles' original proof of Fermat's Last Theorem in the mid-'90s, which was flawed. There was a mistake that he himself discovered and it took him a year to fill the gap. But there is a feeling in the mathematical community that it's possible to actually, eventually, have computers to verify proofs, because it's within this very rigid system. We know for a fact that the Pythagoras Theorem is true. We don't have to believe it, we just know it, right? And that's a very interesting aspect of mathematical knowledge. It means the same thing to everybody. And even if Pythagoras himself hadn't discovered it, someone else would have discovered it, exactly the same thing, which we can't say of practically anything else in life. If Leo Tolstoy hadn't lived, no one would have written '*Anna Karenina*'.

But if Pythagoras did not live, then we'd still have the Pythagoras Theorem. So, what is it about math? Why is it that there are these truths...? Why is it that there are these mathematical truths that we somehow all share, which are persistent, and inevitable, unlike anything else? What does it mean? I think it's a big question that hasn't really been understood. We've only scratched the surface. We don't understand what mathematical knowledge is really about, where it comes from, and how we have access to it. And I think as we learn more about it, we will learn so much more about the physical world, and our consciousness.

MP Right. You certainly fall into a group, I would say, of mathematical optimists. But I can think of two books, one is Janna Levin's first book, as well as David Foster Wallace's book about infinity and they both start out the same way, talking about all of the mathematicians that have driven themselves crazy, or driven themselves to either suicide, or insanity. And they're big names.

EF That's right. With mathematics, you'll never know whether you'll be able to prove something. Pierre Fermat left a note in this old book in 1637, that said, "I found a beautiful proof of this," in what we now call Fermat's Last Theorem, he said, "But this margin is too small to contain it." And it took 350 years to find an actual proof. In my book I talk about it in personal terms, describing my experience solving my first mathematical problem, and wondering will I be able to do it? Am I cut out to be a mathematician? I don't know. Right? This was my first threshold. Will I be able to cross it? There's a fear of not being able to do

Opposite: Ben Berlow, *Untitled*, 2013. Acrylic and gouache on paper. 15 x 20 in.
Courtesy of Rawson Projects, New York.



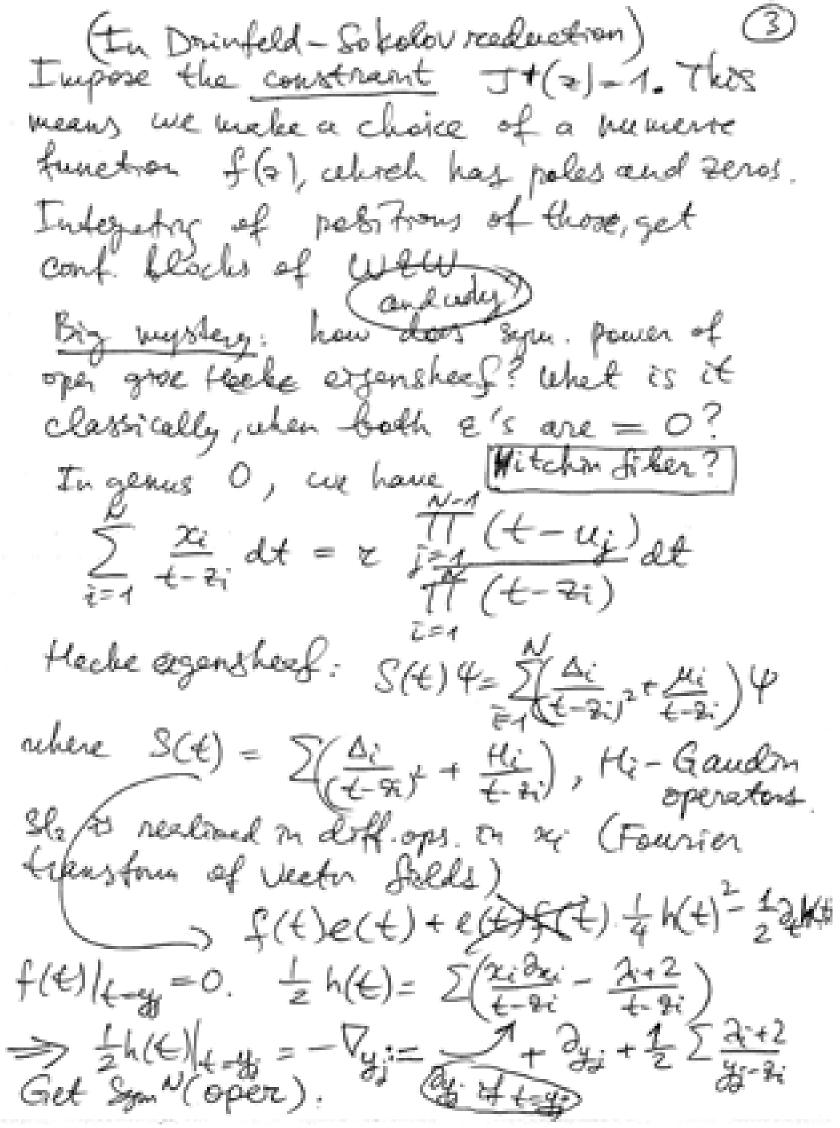
it and nobody can tell you, whether you can or cannot. In other areas of human endeavor you can always sort of, bend the rules. You can always say, "What does it mean to be successful? What does it mean to succeed on a given project?" In order to, you know, improve the productivity in a company, what will constitute a success? If we raise it by 20% is that success? 10%, is that success? How do we measure it?

MP You can have a metric for it.

EF You can always, but it's subjective, it depends. But in mathematics, it's very clear what constitutes the answer. You can spend years and years banging your head against a wall, trying to solve a particular problem, and may not be able to do it, because maybe some crucial ideas just aren't understood. Too soon, too early.

MP Writing a book is a big break, right? Was it helpful, or was it a distraction?

EF Very helpful. I learned so much, through the process of writing this book. The problem is that when I say the word, "Mathematics", most people think of something else, and not what I think of as math. The analogy I make is to imagine an art class, in which all the teacher did was show you how to paint a wall, paint a fence, and told you that was what art was about, and never showed you the paintings of the great masters. Never told you there are museums where you can see them. Then, of course, years later, when people say, "What do you think of art?" you'll say, "Oh, I hate art. I was so bad at it at school." Unfortunately, that's what happens with mathematics, that people say, "I hate math. I'm bad at math." But they're really saying, "I'm bad at painting a fence."



(In Drinfeld–Sokolov reduction) 3

Impose the constraint $J^+(z) = 1$. This means we make a choice of a meromorphic function $f(z)$, which has poles and zeros. Integrating of positions of those, get conf. blocks of ~~WZW~~ and sudy?

Big mystery: how does sym. power of oper give Hecke eigensheaf? What is it classically, when both ε's are = 0? $\boxed{\text{Hitchin fiber?}}$

In genus 0, we have

$$\sum_{i=1}^{N} \frac{x_i}{t-z_i}\,dt = x\,\frac{\prod_{j=1}^{N-1}(t-u_j)}{\prod_{i=1}^{N}(t-z_i)}\,dt$$

Hecke eigensheaf: $S(t)\Psi = \sum_{i=1}^{N}\left(\frac{\Delta_i}{(t-z_i)^2} + \frac{\mu_i}{t-z_i}\right)\Psi$

where $S(t) = \sum\left(\frac{\Delta_i}{(t-z_i)^2} + \frac{H_i}{t-z_i}\right)$, H_i – Gaudin operators.

sl_2 is realized in diff. ops. in x_i (Fourier transform of vector fields)

$f(t)e(t) + e(t)\cancel{f(t)}\ \frac{1}{4}h(t)^2 - \frac{1}{2}\partial_t h(t)$

$f(t)|_{t=y_j} = 0$. $\frac{1}{2}h(t) = \sum\left(\frac{x_i\partial_{x_i}}{t-z_i} - \frac{\lambda_i+2}{t-z_i}\right)$

$\Rightarrow \frac{1}{2}h(t)|_{t=y_j} = -\nabla_{y_j} := \partial_{y_j} + \frac{1}{2}\sum\frac{\lambda_i+2}{y_j-z_i}$ $\quad \partial_{y_j}$ if $t=y_j$

Get $\mathrm{Sym}^N(\text{oper})$.

MP That's right. Kids will struggle with arithmetic, and then think, because they can't calculate four digit numbers in their head, that they're obviously not mathematically inclined, and yet those are very different things.

EF Exactly. There is actually a vast archipelago of knowledge that is completely hidden from the public view. Professional mathematicians know about it but we don't have time to talk about it. It's almost like we are working at this gold mine, digging something beautiful, but we are so tired at the end of the day, because you have to go through a lot of dirt, that you don't have time to step back, look and admire, and show it to others.

Above: Writing from Edward Frenkel's notebook.
Opposite: Ketuta Alexi-Meskhishvili, *Ediso Rising*, 2013. Ilfloflex print.

MP So this reminds you to look at the big picture?

EF Yes. Writing this book made me look at the big picture. It made me think about what mathematics is really about. And also, in what sense there is a link between mathematics and love.

MP What is this link?

EF The link is this idea that I talked about earlier, of universality of mathematical knowledge—that I can meet somebody from a different culture, they may not speak the same language, yet we share all mathematical knowledge just by virtue being humans. I share this with Pythagoras who lived 2500 years ago. Mathematics is the great connector. At the end of the book, I quote Newton, who said that he felt like a little boy on the seashore, playing with pebbles, trying to find a better pebble, or shell, while this vast ocean of knowledge lay beneath him. And it's like we are all like children playing with this stuff, and I want everyone to awaken to this reality that belongs to all of us. It may be the foundation for loving the world, loving each other, because we already have something in common. A mathematical formula doesn't explain love. But it can carry love, can be charged with love.

MP That's beautiful. I always think about the fact that mathematicians still talk about beauty as being important. For a mathematician, beauty is a statement of quality—it remains pure enough that we can call it beautiful. But in art, beauty is cliché—we can no longer call a contemporary piece of art beautiful. We have to find some other way to describe it.

EF It may be beautiful to one, but it might not be beautiful to another. With mathematics, it's much more...

MP What I'm saying is, that even society has said, "We're not supposed to be creating 'beautiful' art anymore." It's not about beauty. Art is not about beauty.

EF But mathematicians have kept this tradition of paying attention to beauty and elegance.

MP Exactly. Elegance – Yes. Elegance and beauty is something that is foundational.

EF It's a guiding principle.