

# QUALIFYING EXAM SYLLABUS

ALEX FINK

*Date of exam:* 26 August 2008.

*Committee:* Rob Kirby, Bernd Sturmfels, David Eisenbud, Satish Rao (EECS), Federico Ardila (SF State).

## 1. MAJOR TOPIC: ALGEBRAIC COMBINATORICS, WITH EMPHASIS ON MATROIDS (ALGEBRA)

Definitions of a matroid: independent sets, bases, circuits, rank functions, closures, the greedy algorithm. Equivalence of these definitions. Linear matroids, hyperplane arrangements, graphical matroids, transversal and cotransversal matroids. Lattice of flats of a matroid, bijection between simple matroids and geometric lattices, the Möbius function, Möbius inversion and its applications. Duality, minors, contraction and deletion. Characteristic polynomials, Tutte polynomials and their applications, internal and external activity. The polytopal view of matroids. Matroid shellability, matroid homology.

Generating functions, quasipolynomials. Polytopes, the Ehrhart quasipolynomial. Posets, lattices, inclusion-exclusion, Möbius function and its applications, the incidence algebra, properties of posets: (semi)modular, distributive, Eulerian, Cohen-Macaulay, geometric.

*References.*

Oxley, *Matroid Theory*, ch. 1, 2.1, 3.1.

White, *Matroid Applications*, ch. 7.

Stanley, *Enumerative Combinatorics*, ch. 2 §1–2, ch. 3, ch. 4 §1–4.

## 2. MAJOR TOPIC: COMMUTATIVE ALGEBRA (ALGEBRA)

Localization, tensor products, prime avoidance, Nakayama's lemma, associated primes and primary decomposition, integral dependence, going-up and going-down theorems, normal domains, flat modules, local criterion for flatness, complete local rings, Hensel's lemma, Hilbert's Nullstellensatz, Krull dimension, Noether normalization, Hilbert-Samuel polynomials, elimination theory, Gröbner bases, Buchberger's algorithm.

*References.*

Eisenbud, *Commutative Algebra with a View toward Algebraic Geometry*, ch. 2–4, 6–9, 12–15.

## 3. MINOR TOPIC: ALGEBRAIC TOPOLOGY (GEOMETRY)

CW complexes, homotopy, homotopy equivalence, fundamental group, Van Kampen's theorem, covering spaces. Cellular homology, simplicial homology, singular homology. Long exact sequences in homology, relative homology, excision, the Mayer-Vietoris sequence. Cohomology, universal coefficients for homology and cohomology, the Künneth formula, cup product, Poincaré duality.

*References.*

Hatcher, *Algebraic Topology*, ch. 1–3.