

# Qualifying Exam Syllabus

Carlos Esparza

**Date:** 05/04/2022

**Room:** TBD

**Committee:** Vera Serganova (Exam Chair), Song Sun (Advisor), Richard Bamler, Alexander Givental (Academic Senate Representative)

## **Major topic: Complex Geometry (Geometry)**

**References:** Huybrechts, *Complex Geometry*, Sections 1-5

Demailly, *Complex Analytic and Differential Geometry*, Section II.8

Roe, *Elliptic Operators, Topology and Asymptotic Methods*, Chapter 13

- Local theory, sheaf, Dolbeault and de Rham cohomology
- Intersection product, Poincaré duality
- Holomorphic vector bundles, Hermitian metrics, connections, curvature
- Hodge theorem, Kodaira–Serre duality
- Kähler manifolds, Kähler identities, Hodge decomposition, Lefschetz decomposition
- Divisors and line bundles, Chern classes of line bundles
- Kodaira vanishing theorem, Lefschetz hyperplane theorem, Lefschetz theorem on  $(1, 1)$ -classes
- Remmert’s proper mapping theorem, Chow’s theorem
- Kodaira embedding theorem, blowing up
- Hirzebruch–Riemann–Roch theorem (assuming Atiyah–Singer formula)

## **Major topic: Riemannian Geometry (Geometry)**

**References:** do Carmo, *Riemannian Geometry*, Chapters 0-5, 7, 9-11

Cheeger–Ebin, *Comparison Theorems in Riemannian Geometry*, Section 1.13

- Smooth manifolds; Maps between smooth manifolds; vector fields; Riemannian metrics
- Affine connections, Levi-Civita connection
- Geodesics, exponential map, normal neighborhoods

- Curvature (Riemann, Ricci, scalar, sectional)
- Hopf–Rinow theorem, Cartan–Hadamard theorem
- Jacobi fields, conjugate points, first and second variations of energy, Bonnet–Myers theorem, Synge–Weinstein theorem
- Index lemma, Rauch comparison theorem
- Cartan–Ambrose–Hicks theorem

## **Minor topic: Lie Theory (Algebra)**

**References:** [Berkeley lecture notes](#), Sections 4.2, 5, 6

Fulton–Harris, *Representation Theory*, Chapters 7-9, 14, 21, 24-26

- Lie groups, Lie algebras
- Semisimple Lie algebras, Killing form, Cartan’s criterion, complete reducibility, root space decomposition
- Root systems, Weyl group, Dynkin diagram, Classification
- Representation theory of semisimple Lie algebras: Weights, Weyl character formula
- Hopf algebras, Semisimple algebraic groups, compact real forms