

MATH 110 Lecture Notes 21

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Theorem. Let $T : V \rightarrow W$ be such that $T^t : W^* \rightarrow V^*$ is onto. Then T is one-to-one.

Proof. Suppose $x \in N(T)$ is nonzero. Then let $f \in V^*$ be such that $f(x) = 1$. Then f does not factor through T , so f is outside $R(T^t)$.

Theorem. Let $T : V \rightarrow W$ be such that $T^t : W^* \rightarrow V^*$ is one-to-one. Then T is onto.

Proof. Suppose $x \in W$ lies outside $R(T)$. Then there exists $f \in W^*$ such that $R(T) \subseteq N(f)$ and $f(x) = 1$. Then $f \neq 0$ but $T^t(f) = fT = 0$.