MATH 110 Lecture Notes 4

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1 More Linear Transformations

Let W_1 and W_2 be subspaces of a vector space V. Then we write $V = W_1 \oplus W_2$ if $V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$. This implies that each element of V can be *uniquely* written in the form $w_1 + w_2$, where $w_1 \in W_1$ and $w_2 \in W_2$. Thus the function $T : V \to V$ given by $T(w_1 + w_2) = w_1$ is well-defined, and we call this function the projection onto W_1 along W_2.

Exercise 2.1.26(a). Prove that T is linear, and that W_1 is the set of fixed points of T.

(b) Prove that $W_1 = R(T)$ and $W_2 = N(T)$.

(c) Describe T if $W_1 = V$.

(d) Describe T if $W_1 = \{0\}$.

Exercise 2.1.38. Let $T : \mathbb{C} \to \mathbb{C}$ be the function defined by $T(z) = \overline{z}$. Prove that T is additive but not $\mathbb{C}$-linear.

Exercise 2.1.40. Let V be a vector space and W be a subspace of V. Define the mapping $\eta : V \to V/W$ by $\eta(v) = v + W$ for $v \in V$.

(a) Prove that η is a linear transformation from V onto V/W and that $N(\eta) = W$.

(b) Suppose that V is finite-dimensional. Use (a) and the dimension theorem to derive a formula relating $\dim V$, $\dim W$, and $\dim(V/W)$.