1. Answer the following statements with either "true" or "false."

   (a) Let A and B be $n \times n$ matrices. Then $(AB)^T = A^T B^T$.
   False. Since $(AB)^T = B^T A^T$, any two non-commuting square matrices work as a counterexample.

   (b) Let A be a symmetric $n \times n$ matrix. Then $\det A \geq 0$.
   False. Consider $-I$ for any odd n.

   (c) Let y_1 and y_2 be linearly independent solutions to a differential equation $y'' + p(t)y' + q(t)y = 0$, where p and q are continuous on $\mathbb{R}$, and suppose $y_1''(0) = 0$. Then $y_2''(0) \neq 0$.
   False. Consider the differential equation $y'' = 0$ which has linearly independent solutions 1 and t.

2. (a) Give the general solution to the following system of differential equations.

$$\mathbf{x}'(t) = \begin{pmatrix} 1 & 9 \\ 1 & 1 \end{pmatrix} \mathbf{x}(t) \tag{1}$$

The characteristic polynomial of the matrix, which we will call A, is

$$\begin{vmatrix} \lambda - 1 & -9 \\ -1 & \lambda - 1 \end{vmatrix} = \lambda^2 - 2\lambda + 1 - 9 = (\lambda - 4)(\lambda + 2).$$

The eigenspace for $\lambda = 4$ is

$$NS(A - 4I) = NS \begin{pmatrix} -3 & 9 \\ 1 & -3 \end{pmatrix} = \text{Span} \left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\},$$

and the eigenspace for $\lambda = -2$ is

$$NS(A + 2I) = NS \begin{pmatrix} 3 & 9 \\ 1 & 3 \end{pmatrix} = \text{Span} \left\{ \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right\}.$$

Therefore the general solution is

$$\mathbf{x}(t) = c_1 e^{4t} \begin{pmatrix} 3 \\ 1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$

   (b) Which type of equilibrium point is the origin for the system (1)? Circle one.

   - saddle point