Solve the following initial value problems.

1.

$$\begin{aligned} 4y'' - 4y' + y &= 0 \\ y(0) &= 1 \\ y'(0) &= \frac{3}{2} \end{aligned}$$

The characteristic equation is $0 = 4\lambda^2 - 4\lambda + 1 = (2\lambda - 1)^2$. Therefore the general solution is

$$y = c_1 e^{t/2} + c_2 t e^{t/2}$$

and

$$y' = \frac{c_1}{2} e^{t/2} + c_2 \left(1 + \frac{t}{2}\right) e^{t/2}.$$

Therefore $1 = c_1$ and $\frac{3}{2} = \frac{c_1}{2} + c_2$, so that $c_1 = c_2 = 1$. Thus the solution to the initial value problem is

$$y = e^{t/2} + t e^{t/2}.$$

2.

$$\begin{aligned} y'' - y' - 12y &= 0 \\ y(0) &= 0 \\ y'(0) &= 1 \end{aligned}$$

The characteristic equation is $0 = \lambda^2 - \lambda - 12 = (\lambda - 4)(\lambda + 3)$, so the general solution is

$$y = c_1 e^{4t} + c_2 e^{-3t}$$

so that

$$y' = 4c_1 e^{4t} - 3c_2 e^{-3t}.$$

Then $0 = c_1 + c_2$ and $1 = 4c_1 - 3c_2$, so that $c_1 = \frac{1}{7}$ and $c_2 = -\frac{1}{7}$. Therefore the solution to the initial value problem is

$$y = \frac{e^{4t} - e^{-3t}}{7}.$$