1. Find matrices S and J such that J is in Jordan canonical form and $A = SJS^{-1}$.

$$A = \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}$$

First we should compute the characteristic polynomial of the matrix:

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - 3 & -1 \\ 1 & \lambda - 3 \end{vmatrix} = (\lambda - 3)^2 + 1 = (\lambda - 3 - i)(\lambda - 3 + i).$$

The eigenspace for $\lambda = 3 + i$ is

$$NS(A - 3I - iI) = NS\begin{pmatrix} -i & 1 \\ -1 & -i \end{pmatrix} = \text{Span}\left\{\begin{pmatrix} 1 \\ i \end{pmatrix}\right\}.$$

The eigenspace for $\lambda = 3 - i$ is

$$NS(A - 3I + iI) = NS\begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix} = \text{Span}\left\{\begin{pmatrix} i \\ 1 \end{pmatrix}\right\}.$$

Therefore we can let

$$J = \begin{pmatrix} 3+i & 0 \\ 0 & 3-i \end{pmatrix}$$

and

$$S = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}.$$

2. Find matrices S and J such that J is in Jordan canonical form and $A = SJS^{-1}$.

$$A = \begin{pmatrix} 3 & 0 \\ -1 & 3 \end{pmatrix}$$

The characteristic polynomial is

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - 3 & 0 \\ 1 & \lambda - 3 \end{vmatrix} = (\lambda - 3)^2.$$

The eigenspace for $\lambda = 3$ is

$$NS(A - 3I) = NS\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} = \text{Span}\left\{\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}.$$

Therefore we can let $\mathbf{v}_2 = (1,0)^T$ and $\mathbf{v}_1 = (A - 3I)\mathbf{v}_2 = (0,-1)^T$. Then

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

and

$$J = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}.$$