1. Let $V = \{(x,y,z) \in \mathbb{R}^3 \mid 2x - y + z = 0\}$ and $\mathbf{u} = (1,0,1)$. Compute $\text{proj}_V \mathbf{u}$.

   First, we need a basis for V. Any one will do, such as $\{(1,2,0)^T, (0,1,1)^T\}$. Once we have a basis, there are two different approaches we can take.

   (a) Let $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{pmatrix}$. Since A has linearly independent columns, $A^T A$ is invertible, and

   $$\text{proj}_V \mathbf{u} = A(A^T A)^{-1} A^T \mathbf{u} = \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}.$$

   (b) We can apply Gram-Schmidt to our basis to obtain an orthogonal basis for V:

   $$\mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

   $$\mathbf{p}_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - \frac{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

   Since $\mathbf{u} \perp \mathbf{p}_2$,

   $$\begin{aligned} \text{proj}_V \mathbf{u} &= \text{proj}_{\mathbf{p}_1} \mathbf{u} + \text{proj}_{\mathbf{p}_2} \mathbf{u} \\ &= \text{proj}_{\mathbf{p}_1} \mathbf{u} \\ &= \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}. \end{aligned}$$

2. Let $T : P_2 \to \mathbb{R}^2$ be defined by

   $$T(p) = \begin{pmatrix} p(0) \\ p'(1) \end{pmatrix}.$$

   Let $B = \{1, x-1, (x-1)^2\}$ and C be the standard basis on $\mathbb{R}^2$. Compute $[T]_{BC}$.

   First we need to apply T to the elements of B:

   $$\begin{aligned} T(1) &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ T(x-1) &= \begin{pmatrix} -1 \\ 1 \end{pmatrix} \\ T((x-1)^2) &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned}$$

Since C is the standard basis, each of these vectors is equal to its own coordinate vector relative to C. Therefore

$$[T]_{BC} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$