1. Consider the inner product on P_2 given by

$$\langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1).$$

Let $f_1(x) = 1$ and $f_2(x) = x^2$. Using this inner product, compute $\text{proj}_{f_2} f_1$.

$$\text{proj}_{f_2} f_1 = \frac{\langle f_1, f_2 \rangle}{\langle f_2, f_2 \rangle} f_2 = \frac{2}{2} \cdot x^2 = x^2.$$

2. Find the line which best fits the data $(0, 1)$, $(1, 0)$, and $(2, 1)$ in the least-squares sense.

Let $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Then

$$A^T A = \begin{pmatrix} 3 & 3 \\ 3 & 5 \end{pmatrix}$$

and

$$A^T \mathbf{b} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}.$$

Therefore, the equation $A^T A\mathbf{x} = A^T\mathbf{b}$ can be solved by row reducing the augmented matrix

$$\begin{pmatrix} 3 & 3 & 2 \\ 3 & 5 & 2 \end{pmatrix}$$

which row reduces to

$$\begin{pmatrix} 3 & 3 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 3 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & 0 \end{pmatrix}$$

Therefore the best fit line is $y = \frac{2}{3}$.