1. Find a basis for $NS(A)$, where
$$A = \begin{pmatrix} 3 & 4 & 1 & -1 & 3 \\ 1 & 1 & 1 & 0 & 1 \\ 2 & 2 & -2 & 2 & -2 \end{pmatrix}.$$

   The matrix row reduces as follows:
$$\begin{pmatrix} 0 & 1 & -2 & -1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & -4 & 2 & -4 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & 2 & -1 & 2 \end{pmatrix}$$
$$\begin{pmatrix} 2 & 0 & 6 & 2 & 2 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & 2 & -1 & 2 \end{pmatrix}$$
$$\begin{pmatrix} 2 & 0 & 0 & 5 & -4 \\ 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & 2 & -1 & 2 \end{pmatrix}$$

   Thus we can think of the last two columns as corresponding to free variables. If we parametrize these free variables by $2s$ and t, we get
$$NS(A) = \left\{ \begin{pmatrix} -5s+2t \\ 4s-2t \\ s-t \\ 2s \\ t \end{pmatrix} : s,t \in \mathbb{R} \right\} = \text{Span} \left\{ \begin{pmatrix} -5 \\ 4 \\ 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

   Therefore a basis is given by $\{(-5,4,1,2,0)^T, (2,-2,-1,0,1)^T\}$.

2. Find a basis for the subspace $T = \{f \in P_3 \mid f'(2) = 0\}$ of P_3.

   Let $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$. Then $f'(x) = a_1 + 2a_2x + 3a_3x^2$, and $f \in T$ if and only if
$$0 = f'(2) = a_1 + 4a_2 + 12a_3,$$
   or if $a_1 = -(4a_2 + 12a_3)$. Therefore
$$T = \{a_0 - (4a_2 + 12a_3)x + a_2x^2 + a_3x^3 \mid a_0, a_2, a_3 \in \mathbb{R}\}$$
   and a basis for T is given by
$$\{1, x^2 - 4x, x^3 - 12x\}.$$

   Alternatively, we can start with the basis $\{1, x-2, (x-2)^2, (x-2)^3\}$ of P_3. The three polynomials 1, $(x-2)^2$, and $(x-2)^3$ are in T, so they must form a basis of T since they are linearly independent and $\dim T \leq 3$.