1. Let $S = \{(x, y, z) \in \mathbb{R}^3 \mid x - y + z \in \mathbb{Z}\}$.

   (a) Is S closed under addition?
   Yes. Let $(x_1, y_1, z_1), (x_2, y_2, z_2) \in S$. Then their sum, $(x_1 + x_2, y_1 + y_2, z_1 + z_2)$, is in S since

   $$(x_1 + x_2) - (y_1 + y_2) + (z_1 + z_2) = (x_1 - y_1 + z_1) + (x_2 - y_2 + z_2) \in \mathbb{Z}.$$

   (b) Is S closed under scalar multiplication?
   No. Consider $\frac{1}{2} \cdot (1, 0, 0)$.

   (c) Is S a subspace of $\mathbb{R}^3$?
   No.

2. Let $T = \{f \in P_3 \mid f'(2) = 0\}$.

   (a) Is T closed under addition?
   Yes. Let $f, g \in T$. Then $f + g \in P_3$ and

   $$(f + g)'(2) = f'(2) + g'(2) = 0.$$

   (b) Is T closed under scalar multiplication?
   Yes. Let $f \in T$ and $r \in \mathbb{R}$. Then $rf \in P_3$ and

   $$(rf)'(2) = r(f'(2)) = 0.$$

   (c) Is T a subspace of P_3?
   Yes.