

1. Answer each of the following statements with “true” or “false.”

(a) If A and B are invertible $n \times n$ matrices, then AB is invertible and

$$(AB)^{-1} = A^{-1}B^{-1}.$$

False, $(AB)^{-1} = B^{-1}A^{-1}$. Any two invertible matrices which do not commute will provide a counterexample.

(b) If A and B are invertible $n \times n$ matrices, then $A + B$ is invertible as well.

False. Consider $I + (-I) = 0$.

(c) Any line through the origin in \mathbb{R}^3 is equal to $\text{Span}\{\mathbf{u}\}$ for some nonzero $\mathbf{u} \in \mathbb{R}^3$.

True.

2. Find the inverse of the following matrix.

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & -1 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

The inverse can be obtained by row reduction.

$$\begin{pmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & -2 & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 2 & -3 & -1 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 2 & -3 & -1 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{pmatrix}$$

Therefore $A^{-1} = \begin{pmatrix} -1 & 1 & 1 \\ 2 & -3 & -1 \\ 2 & -2 & -1 \end{pmatrix}$.