

1. Find all solutions to the following system of equations.

$$\begin{aligned}x + z &= 6 \\w - x + y &= 0 \\2w - 3x - z &= 1\end{aligned}$$

After rearranging the equations, this system gives the augmented matrix

$$\begin{pmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 6 \\ 2 & -3 & 0 & -1 & 1 \end{pmatrix}$$

which row reduces in the following way:

$$\begin{aligned}&\begin{pmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 6 \\ 0 & -1 & -2 & -1 & 1 \end{pmatrix} \\&\begin{pmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 6 \\ 0 & 0 & -2 & 0 & 7 \end{pmatrix} \\&\begin{pmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 6 \\ 0 & 0 & 1 & 0 & -7/2 \end{pmatrix} \\&\begin{pmatrix} 1 & -1 & 0 & 0 & 7/2 \\ 0 & 1 & 0 & 1 & 6 \\ 0 & 0 & 1 & 0 & -7/2 \end{pmatrix} \\&\begin{pmatrix} 1 & 0 & 0 & 1 & 19/2 \\ 0 & 1 & 0 & 1 & 6 \\ 0 & 0 & 1 & 0 & -7/2 \end{pmatrix}\end{aligned}$$

Therefore  $z$  is a free variable,  $y = -\frac{7}{2}$ ,  $x = 6 - z$ , and  $w = \frac{19}{2} - z$ .

2. Let  $A = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & 0 & 3 \\ 0 & -1 & -2 \end{pmatrix}$ .

- (a) Is  $AB$  defined? If so, compute it.

It is defined, since the number of columns of  $A$  is equal to the number of rows of  $B$ , and

$$AB = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 3 \\ 0 & -1 & -2 \end{pmatrix} = \begin{pmatrix} 4 & -1 & 4 \\ 2 & 1 & 5 \end{pmatrix}.$$

- (b) Is  $BA$  defined? If so, compute it.

No, since  $B$  has three columns but  $A$  only has two rows.