

# MATH 54 Lecture Notes 24

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## 1 Euler's Formula and Magnitude

Let  $\alpha, \beta \in \mathbb{R}$ . Then

$$|e^{\alpha+\beta i}| = e^\alpha |\cos \beta + i \sin \beta| = e^\alpha \sqrt{\cos^2 \beta + \sin^2 \beta} = e^\alpha.$$

Also note that

$$e^{2\pi i} = \cos(2\pi) + i \sin(2\pi) = 1.$$

Therefore the exponential function is periodic with period  $2\pi i$ .

## 2 Sources, Sinks, etc.

Consider a differential equation

$$\mathbf{x}'(t) = A\mathbf{x}(t)$$

where  $A$  is a diagonalizable  $2 \times 2$  matrix with real entries. Several things can happen at the origin:

- Source. The eigenvalues of  $A$  are real and positive, so solutions move away from the origin as  $t \rightarrow \infty$ . Exercise 7.5.6.
- Sink. The eigenvalues of  $A$  are real and positive, so solutions move towards the origin as  $t \rightarrow \infty$ . Exercise 7.5.5. Do the umbrella demonstration here. The hanging umbrella is a sink; the balanced umbrella is a source.
- Saddle point. The eigenvalues of  $A$  are real with opposite signs. Exercise 7.5.3.
- Spiral in. The eigenvalues of  $A$  are complex with negative real part. Exercise 7.6.2.
- Spiral out. The eigenvalues of  $A$  are complex with positive real part. Exercise 7.6.4.
- Periodic. The eigenvalues of  $A$  are purely imaginary. Exercise 7.6.6.