

MATH 54 Midterm 2

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Each problem is worth 30 points. Please show your work, except for problem 1, where only the answer is necessary.

Name:

Student ID:

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1. Answer the following statements with “true,” “false,” or “I don’t know.” Five points will be given for the correct answer, and three points will be given for each “I don’t know” answer.

(a) For any $c \in \mathbb{R}$ and any $\mathbf{v} \in \mathbb{R}^n$, $\|c\mathbf{v}\| = c\|\mathbf{v}\|$.

(b) Let Q be an orthogonal $n \times n$ matrix with all its entries in \mathbb{R} . Then $|\det Q| = 1$.

(c) Let A and B be two $n \times n$ matrices such that B is obtained from A by interchanging two rows. Then $\det A + \det B = 0$.

(d) The following matrix is diagonalizable, with all its eigenvalues being real numbers.

$$B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

(e) Let A be a nilpotent matrix. That is, $A^n = 0$ for some positive integer n . Then $A + I$ is invertible.

(f) Let J_1 and J_2 be two $n \times n$ matrices in Jordan canonical form. Then $J_1 + J_2$ is also in Jordan canonical form.

2. Give matrices S and J such that J is in Jordan canonical form and $A = SJS^{-1}$.

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 2 & -2 & -2 \\ 4 & -8 & -2 \end{pmatrix}$$

3. Give matrices S and J such that J is in Jordan canonical form and $A = SJS^{-1}$.

$$A = \begin{pmatrix} 4 & 1 & -1 \\ 4 & 3 & -4 \\ 5 & 1 & -2 \end{pmatrix}$$

4. Consider the following subspace of P_2 .

$$S = \{a + ax + bx^2 \mid a, b \in \mathbb{R}\}$$

(a) Give an orthogonal basis for S under the following inner product.

$$\langle f, g \rangle = f(-1)g(-1) + f(0)g(0) + f(1)g(1)$$

- (b) Let $T : P_2 \rightarrow P_2$ be the orthogonal projection of P_2 onto S with respect to the inner product from part (a). Let $C = \{1, 1+x, (1+x)^2\}$. Give the matrix for T relative to the ordered basis C of P_2 .

(c) Give the projection of $h(x) = \cos(\pi x)$ onto S with respect to the inner product from part (a).

5. Consider the following subspace of P_3 .

$$S = \{f \in P_3 \mid f'(1) = 0\}$$

(a) Give a basis for S .

(b) Out of the functions in S , give the one that best fits the data $(0, 0)$, $(1, 0)$, $(2, 8)$, and $(-1, 2)$ using the least squares method. Note that the answer to this question must be a *polynomial* of degree 3 or less.