

1. Answer the following “true” or “false.”

(a) Let $T : C[-2, 2] \rightarrow \mathbb{R}^2$ be defined by

$$T(f) = \begin{pmatrix} f(-1) \\ f(1) \end{pmatrix}.$$

Then T is a linear transformation.

(b) Let $T(A) = BAC$, for two fixed matrices B and C . Then T is a linear transformation.

(c) Let N be the set of nilpotent $n \times n$ matrices. (A matrix is nilpotent if some power of it is the zero matrix.) Then N is a linear vector space.

(d) Let A and B be two $n \times n$ matrices. Then $rk(AB) \leq rk(B)$.

2. Compute A^{-1} .

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 3 & 1 \\ 0 & 0 & -2 \end{pmatrix}$$

3. Find a basis for $NS(A)$.

$$A = \begin{pmatrix} 1+i & 2 & -i \\ 1 & 3 & 1+i \\ 2 & 2-2i & -1-i \end{pmatrix}$$

4. Consider \mathbb{C}^2 as a vector space over \mathbb{R} . Find a basis for the following subspace of \mathbb{C}^2 .

$$S = \{(z_1, z_2) \in \mathbb{C}^2 \mid z_1 = \overline{z_2}\}$$