

1. Answer the following questions with “true” or “false.”
 - (a) Let A be an $n \times n$ matrix such that $A^2 + A - 5I = 0$. Then A is invertible.
 - (b) Let $T(x, y, z) = (x - z + 1, y - z, z)$. Then T is a linear transformation.
 - (c) Let A be an $n \times n$ matrix such that all the diagonal entries of A are 0. Then A is singular.
 - (d) Let A be an $n \times n$ matrix such that $rk(A) = n$ and $A^2 = A$. Then $A = I$.
 - (e) Let $S = \{f \in C^\infty[-1, 1] \mid f'(0) = 0\}$. Then S is a subspace of $C^\infty[-1, 1]$.
2. Give a basis for $NS(A)$ and a basis for $RS(A)$.

$$A = \begin{pmatrix} 1+i & 2i & -3+5i & 6i \\ 2i & -1 & -4-i & -4+2i \\ i & -3 & -7-3i & -7+i \\ i & 0 & -1 & -1+i \end{pmatrix}$$

3. Find all solutions.

$$\begin{aligned} 4x - 3y + z &= 3 \\ x + y - z &= 0 \end{aligned}$$

4. For $f \in C^\infty[a, b]$, define $D^2f = D(Df)$, where Df refers to the derivative of f .
 - (a) Is D^2 a linear transformation?
 - (b) Give a basis for the null space of D^2 .
5. Give a 3×3 matrix that rotates the xz -plane by 60 degrees and leaves the y -axis fixed.