MATH 54 Midterm 1

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Each problem is worth 30 points. Please show your work, except for problem 1, where only the answer is necessary. For any problems that involve row reducing matrices, give the matrix that you get after *each elementary row operation*.

Name:

Student ID:

Total	
1	
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1. Answer the following statements with "true," "false," or "I don't know." Five points will be given for the correct answer, and three points will be given for each "I don't know" answer.

   (a) Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = z - \overline{z}$ for all $z \in \mathbb{C}$. Then f is a $\mathbb{C}$-linear map.

   (b) Let A be a 2×2 matrix such that $rk(A) = 1$. Then $rk(A^2) = 1$.

   (c) Let U, V, and W be vector spaces. Let $f : U \to V$ and $g : V \to W$ be linear transformations. Then $g \circ f : U \to W$ is a linear transformation[1].

[1] The notation $g \circ f$ means f followed by g. That is, for any $\mathbf{u} \in U$, $g \circ f(\mathbf{u}) = g(f(\mathbf{u}))$.

(d) Any system of four linear equations in five unknowns has infinitely many solutions.

(e) Suppose A is an $n \times n$ matrix such that $A^2 = 0$. Then $A + I$ is invertible.

(f) Let $\langle p, q \rangle = p(0)q(0) + p(1)q(1) + p(2)q(2)$ for all $p, q \in P_3$. This defines an inner product on P_3.

2. Consider the following matrix A.

$$A = \begin{pmatrix} 1-i & \frac{1+i}{2} & 0 & 1 \\ -2 & -i & 1 & 2-i \\ -4 & -2i & 3 & 7-2i \end{pmatrix}$$

(a) Give a basis for $NS(A)$.

(b) What is $\dim NS(A)$?

(c) What is $rk(A)$?

3. Find all solutions.

$$\begin{aligned} 2x + y + 4z &= 5 \\ x + 3y + 7z + w &= 2 \\ y + 2z &= -1 \end{aligned}$$

4. Consider the following subset of P_3.

$$S = \{f \in P_3 \mid f(1) - f(2) = \gamma\}$$

(a) For which values of γ is S a subspace of P_3?

(b) For each of the above values of γ, find a basis for S.

5. Consider the following two matrices.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & -3 \\ 1 & -1 & 2 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{pmatrix}$$

(a) Compute A^{-1}.

(b) Compute B^{-1}.

(c) Compute $(AB)^{-1}$.

(d) Compute $(BA)^{-1}$.