1. Are each of the following functions piecewise continuous on the interval $[-\pi, \pi]$?

   (a) $g(x) = \begin{cases} -\frac{1}{x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

   No, since there is a vertical asymptote.

   (b) $h(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

   Yes. This function is actually continuous on $\mathbb{R}$.

2. Find the Fourier series for the following function, using $L = \pi$.

$$f(x) = \begin{cases} 2 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -2 & \text{if } x < 0 \end{cases}$$

The function f is odd, so $a_m = 0$. We only need to compute b_m, and

$$\begin{aligned} b_m &= \frac{2}{L} \int_0^L f(x) \sin\left(\frac{m\pi x}{L}\right) \, dx \\ &= \frac{4}{\pi} \int_0^\pi \sin(mx) \, dx \\ &= \frac{4}{m\pi}(\cos(0) - \cos(m\pi)) \\ &= \frac{4}{m\pi}(1 - (-1)^m). \end{aligned}$$

Therefore the Fourier series of f is

$$f(x) = \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{1 - (-1)^m}{m} \sin(mx).$$

This can be rewritten as

$$f(x) = \frac{8}{\pi} \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin((2n+1)x).$$