

1. Compute $|e^{1+3i}|$.

$$|e^{1+3i}| = \left| e\sqrt{\cos^2 3 + \sin^2 3} \right| = e$$

2. (a) Give the general solution to the following system of differential equations.

$$\mathbf{x}'(t) = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \mathbf{x}(t) \quad (1)$$

The matrix, which we will call A , is diagonalizable by the Spectral Theorem. First we need its characteristic polynomial.

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - 3 & -1 \\ -1 & \lambda - 3 \end{vmatrix} = \lambda^2 - 6\lambda + 8 = (\lambda - 2)(\lambda - 4)$$

For $\lambda = 2$, the eigenspace is

$$NS(A - 2I) = NS \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \text{Span} \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$$

For $\lambda = 4$, it is

$$NS(A - 4I) = NS \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} = \text{Span} \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}.$$

Therefore the general solution to (1) is

$$\mathbf{x}(t) = c_1 e^{2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

- (b) For the above system of differential equations, is the origin a source, sink, or saddle point?
Both eigenvalues are positive, so it is a source.