1. (a) Compute $W(t, e^t)$.

$$W(t, e^t) = \begin{vmatrix} t & e^t \\ 1 & e^t \end{vmatrix} = (t-1)e^t$$

(b) Can $f_1(t) = t$ and $f_2(t) = e^t$ be solutions to a differential equation $y'' + p(t)y' + q(t)y = 0$ on the interval $(-1, 2)$, where p and q are continuous everywhere?

No. If f_1 and f_2 were solutions to such a differential equation, their Wronskian would have to either be zero on the entire interval or nonzero on the entire interval. However, $W(t, e^t)(1) = 0$ and $W(t, e^t)(0) = -1$.

2. Solve the following initial value problem.

$$\begin{aligned} y'' + 6y' + 9y &= 0 \\ y(0) &= 1 \\ y'(0) &= 2 \end{aligned}$$

The characteristic equation for this differential equation is $\lambda^2+6\lambda+9 = (\lambda+3)^2$. Therefore the general solution is

$$y = c_1 e^{-3t} + c_2 t e^{-3t}$$

so that

$$y' = -3c_1 e^{-3t} + c_2 e^{-3t} - 3c_2 t e^{-3t}.$$

Using the initial conditions to solve for c_1 and c_2, we obtain

$$\begin{aligned} 1 &= c_1 \\ 2 &= -3c_1 + c_2. \end{aligned}$$

The solution to this system of equations is $c_1 = 1$ and $c_2 = 5$. Therefore, the solution to the initial value problem is

$$y = e^{-3t} + 5te^{-3t}.$$