

1. For each of the following statements, write the word “true” or “false.”

(a) Let A and B be two $n \times n$ matrices. Then $\det(A + B) = \det A + \det B$.

False. For a counterexample, consider two nilpotent matrices that add up to an invertible matrix.

(b) Let A and B be two $n \times n$ matrices. Then $\det(AB) = (\det A)(\det B)$.

True.

2. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be given by

$$T(x, y, z) = (x - y + z, -x + 2z, 2x - 2y + z).$$

Give the matrix $[T]_C$, where C is the ordered basis $\{(0, 0, 1), (1, -1, 0), (-1, 0, 1)\}$.

Let B be the standard basis. Then

$$[T]_B = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 0 & 2 \\ 2 & -2 & 1 \end{pmatrix}.$$

Then if

$$P = \begin{pmatrix} 0 & 1 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix},$$

we have that $[T]_C = P^{-1}[T]_B P$. Therefore,

$$[T]_C = \begin{pmatrix} 4 & 5 & 2 \\ -2 & 1 & -3 \\ -3 & -1 & -3 \end{pmatrix}.$$