1. For each of the following statements, write the word "true" or "false."

   (a) Let P be the orthogonal projection matrix for a subspace V of $\mathbb{R}^n$. Then $NS(P - I_n) = V$.
   True. The space $NS(P - I)$ is the set of vectors where $P\mathbf{v} = \mathbf{v}$, which is V.

   (b) Suppose A is an $n \times n$ matrix such that $AA^* = I$. Then, for any $\mathbf{u}, \mathbf{v} \in \mathbb{C}^n$, $\langle A\mathbf{u}, A\mathbf{v}\rangle = \langle \mathbf{u}, \mathbf{v}\rangle$, where the inner product is the standard dot product on $\mathbb{C}^n$.
   True. By the basic properties of the adjoint matrix,
$$\langle A\mathbf{u}, A\mathbf{v}\rangle = \langle \mathbf{u}, A^*A\mathbf{v}\rangle = \langle \mathbf{u}, \mathbf{v}\rangle.$$

2. Find an orthogonal basis for P_2 with respect to the inner product
$$\langle f, g\rangle = \int_0^1 f(x)\overline{g(x)}\, dx.$$

We start with the basis $1, x, x^2$ and apply Gram-Schmidt to it. First let $f_1(x) = 1$. Then
$$f_2(x) = x - \text{proj}_{f_1} x = x - \frac{\int_0^1 x\, dx}{\int_0^1 dx} = x - \frac{1}{2}.$$

Finally,
$$\begin{aligned} f_3(x) &= x^2 - \text{proj}_{f_1} x^2 - \text{proj}_{f_2} x^2 \\ &= x^2 - \frac{\int_0^1 x^2\, dx}{\int_0^1 dx} - \frac{\int_0^1 x^3 - \frac{x^2}{2}\, dx}{\int_0^1 \left(x - \frac{1}{2}\right)^2 dx}\left(x - \frac{1}{2}\right) \\ &= x^2 - \frac{1}{3} - \frac{\frac{1}{4} - \frac{1}{6}}{\frac{1}{3} - \frac{1}{2} + \frac{1}{4}}\left(x - \frac{1}{2}\right) \\ &= x^2 - \frac{1}{3} - \left(x - \frac{1}{2}\right) = x^2 - x + \frac{1}{6}. \end{aligned}$$