1. For each of the following statements, write the word "true" or "false."

   (a) Let $T : \mathbb{R}^2 \to M_{22}$ be defined by

   $$T(a, b) = \begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix}.$$

   Then T is a linear transformation.

   False. The zero vector is not mapped to the zero matrix.

   (b) Let A and B be two $n \times n$ matrices. Then $rk(AB) \leq rk(A)$.

   True. Let $\mathbf{u} \in CS(AB)$. Then there exists $\mathbf{v}$ such that $AB\mathbf{v} = \mathbf{u}$. This also shows that $\mathbf{u} \in CS(A)$. Therefore $CS(AB) \subseteq CS(A)$, so $\dim CS(AB) \leq \dim CS(A)$.

2. Find a basis for the following subspace of P_4.

   $$S = \{f \in P_4 \mid f(1) = f'(0) = 0\}$$

   If $f(1) = 0$, then $(x - 1)$ divides f. If $f'(0) = 0$, then the linear term of f must be zero. Therefore S is the set of polynomials of the form

   $$(ax^3 + bx^2 + cx + d)(x - 1) = ax^4 + (b - a)x^3 + (c - b)x^2 + (d - c)x - d$$

   where the linear term, $(d - c)x$, is zero. Therefore S can be rewritten as the set of polynomials of the form

   $$(ax^3 + bx^2 + cx + c)(x - 1).$$

   A basis is given by the polynomials

   $$x^3(x - 1), x^2(x - 1), x^2 - 1.$$