

1. For each of the following statements, write the word “true” or “false.”
 - (a) Let W be a subspace of a vector space V . Then any basis for W can be expanded to a basis for V .
True.
 - (b) Let W be a subspace of a finite-dimensional vector space V which is not equal to V . Then $\dim W < \dim V$.
True. Otherwise a basis of W could be expanded to a basis for V which would have too many elements.
2. Find a basis for the following subspace of M_{33} , where $\mathbf{u} = (1, 0, -1)^T$.

$$S = \{B \in M_{33} \mid B\mathbf{u} = \mathbf{0}\}$$

All such matrices have the form

$$\begin{pmatrix} a & b & a \\ c & d & c \\ e & f & e \end{pmatrix}.$$

Therefore a basis is given by

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$