

1. For each of the following statements, write the word “true” or “false.”

(a) Let  $T : C[-2, 2] \rightarrow \mathbb{R}^2$  be defined by

$$T(f) = \begin{pmatrix} f(-1) \\ f(1) \end{pmatrix}.$$

Then  $T$  is a linear transformation.

True.

(b) Let  $A$  and  $B$  be two  $n \times n$  matrices. Then  $rk(AB) \leq rk(B)$ .

True. Since  $NS(B) \subseteq NS(AB)$ ,  $rk(B) \geq rk(AB)$ .

2. Consider  $\mathbb{C}^2$  as a vector space over  $\mathbb{R}$ . Find a basis for the following subspace of  $\mathbb{C}^2$ .

$$S = \{(z_1, z_2) \in \mathbb{C}^2 \mid z_1 = \overline{z_2}\}$$

The vector space  $S$  consists of ordered pairs of the form

$$(a + bi, a - bi),$$

so a basis is given by  $(1, 1)$  and  $(i, -i)$ .